ESERCIZI IN PIÙ

ESERCIZI DI FINE CAPITOLO

Semplifica le seguenti espressioni. Supponi positivi i fattori letterali che compongono i radicandi.

1 $\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}+\frac{\sqrt{x}}{x-1}+\frac{1}{\sqrt{x}-1}\right)\cdot\frac{\sqrt{x}+1}{x+2}$ $\left[\frac{\sqrt{x}+1}{x-1}\right]$

2 $\left(\frac{1}{x+\sqrt{x}}+\frac{\sqrt{x}-1}{\sqrt{x}}-\frac{x}{\sqrt{x}+x}+\frac{\sqrt{x}}{\sqrt{x}+1}\right)\cdot\frac{x-1}{x}$ $\left[\frac{\sqrt{x}\,(\sqrt{x}-1)}{x}\right]$

3 $\left(\frac{x}{\sqrt{y}}+\frac{8y}{\sqrt{x}}\right):\frac{x-2\sqrt{xy}+4y}{\sqrt{y}}-1$ $\left[\frac{2\sqrt{xy}}{x}\right]$

4 $\frac{x}{\sqrt{x}+1}-\frac{1}{\sqrt{x}-1}+\frac{x}{1-\sqrt{x}}-\frac{1}{\sqrt{x}+1}$ $\left[\frac{-2x-2\sqrt{x}}{x-1}\right]$

5 $\left[(\sqrt[4]{32x^5y}-\sqrt[4]{2xy^5})\cdot\frac{\sqrt[4]{8xy^3}}{2x-y}\right]^3$ $[8xy^3\sqrt{x}]$

6 $\sqrt[6]{\frac{a+b}{a^3(a-b)}}\cdot\left[\sqrt[3]{\frac{(a-b)^2}{a^2+ab}}:\sqrt{\frac{a-b}{a^2}}\right]:\sqrt[6]{\frac{a^2}{a+b}}$ $\left[\frac{\sqrt[6]{a^5}}{a}\right]$

Semplifica le seguenti espressioni, utilizzando, quando è possibile, le proprietà delle potenze. Supponi che le basi letterali siano positive.

7 $\frac{(x-1)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}+(x+1)^{\frac{1}{2}}}-\frac{(x+1)^{\frac{1}{2}}}{(x+1)^{\frac{1}{2}}-(x-1)^{\frac{1}{2}}}$ $[-x]$

8 $\frac{(x+2)^{\frac{1}{2}}(x^2-4)^{-\frac{1}{2}}}{(x-2)^{-1}(x^2-2x)^{-\frac{3}{2}}}:\left(\frac{(x-2)^{-\frac{1}{2}}}{x^{\frac{1}{2}}}\right)^{-3}$ $\left[(x-2)^{\frac{1}{2}}\right]$

9 $\left[\left(\frac{2x-3}{2x+3}\right)^{-\frac{1}{2}}:\left(\frac{6x+9}{4x^2-9}\right)^{-\frac{1}{3}}\right]^3\cdot\left[\frac{(2x+3)^3}{3^{-2}}\right]^{-\frac{1}{2}}$ $\left[(2x-3)^{-\frac{5}{2}}\right]$

10 $\left[\left(\frac{ab^{\frac{1}{3}}\cdot a^{-\frac{1}{2}}b^{\frac{2}{3}}}{a^{\frac{3}{2}}b^5}\right)^{-\frac{1}{2}}:\left(\frac{a^{\frac{1}{4}}}{b^{-\frac{1}{2}}}\right)^2\right]^6:b^{\frac{11}{2}}$ $\left[b^{\frac{1}{2}}\right]$

11 $\frac{(ab-b^2)^{\frac{1}{3}}}{(a^2+ab)^{\frac{2}{3}}}:\left[\frac{a+b}{(a-b)^2}\right]^{\frac{1}{3}}:\left[\left(\frac{a^2}{b}\right)^{\frac{1}{3}}\cdot\frac{a+b}{a-b}\right]^{-1}$ $[1]$

Risolvi le seguenti equazioni.

12 $\frac{x\sqrt{5}-\sqrt{3}}{x\sqrt{5}+\sqrt{3}}=\frac{5x-3}{5x+3}$ $[0]$

13 $\dfrac{(2\sqrt{2}+1)y}{\sqrt{2}} - \dfrac{y\sqrt{2}}{\sqrt{2}+1} - \dfrac{y}{2+\sqrt{2}} = \sqrt{2}+3$ $[\sqrt{2}+1]$

14 $\dfrac{x\sqrt{2}}{\sqrt{6}+3} + \dfrac{x\sqrt{3}}{2+\sqrt{6}} - \dfrac{2x-\sqrt{2}}{\sqrt{3}} = \dfrac{x-3\sqrt{2}}{\sqrt{2}}$ $[\sqrt{2}+\sqrt{3}]$

15 $\dfrac{1}{3-3\sqrt{3}x} + \dfrac{2\sqrt{3}}{1-3x^2} = \dfrac{\sqrt{3}}{3+3\sqrt{3}x}$ $\left[\dfrac{6-7\sqrt{3}}{3}\right]$

16 $\dfrac{x+3\sqrt{2}}{x^2+2\sqrt{2}x+2} + \dfrac{\sqrt{2}x}{2(x+\sqrt{2})} - \dfrac{\sqrt{2}}{2} = 0$ [impossibile]

17 $\dfrac{4x}{\sqrt{3}x-3} + \dfrac{2\sqrt{3}x-4}{x^2-2\sqrt{3}x+3} = \dfrac{4}{\sqrt{3}}$ $[2(\sqrt{3}-1)]$

Risolvi i seguenti sistemi.

18 $\begin{cases} \sqrt{5}x + y = -4 \\ 2x + 2\sqrt{5}y = 0 \end{cases}$ $[-\sqrt{5}; 1]$

19 $\begin{cases} \sqrt{2}x + 3y = 3\sqrt{2} - 1 \\ x + y = 2\sqrt{2} - 1 \end{cases}$ $[\sqrt{2}; \sqrt{2}-1]$

20 $\begin{cases} \dfrac{2\sqrt{3}x+5y}{3} - \dfrac{y+\sqrt{3}}{2} = \dfrac{5-3\sqrt{3}}{6} \\ \dfrac{\sqrt{3}(x-2y)}{2} - \dfrac{2\sqrt{3}-5y}{3} = \dfrac{2\sqrt{3}-1}{6} \end{cases}$ $[(\sqrt{3}; -1)]$

21 $\begin{cases} \dfrac{x}{\sqrt{2}} + \dfrac{y}{\sqrt{3}} = 2\sqrt{3} \\ 2\sqrt{3}x = \dfrac{1}{3}(10\sqrt{3} - 5\sqrt{2}x) + 2\sqrt{2}y \end{cases}$ $[(\sqrt{6}; 3)]$

Risolvi le seguenti disequazioni.

22 $2bx + \sqrt{5} < \sqrt{5}bx - 3 \quad (b < 0)$ $\left[x < \dfrac{5\sqrt{5}+11}{b}\right]$

23 $\dfrac{x}{a-\sqrt{2}} + \dfrac{1-x}{a+\sqrt{2}} > 0$

$\left[a < -\sqrt{2} \vee a > \sqrt{2}, x > \dfrac{2-\sqrt{2}a}{4}; -2 < a < \sqrt{2}, x < \dfrac{2-\sqrt{2}a}{4}; a = -\sqrt{2} \vee a = \sqrt{2}, \text{perde di significato}\right]$

24 $\dfrac{\sqrt{2}ax + \sqrt{6}}{\sqrt{3}} - \dfrac{2-x}{\sqrt{2}} \leq a\sqrt{2}$

$\left[a < -\dfrac{\sqrt{3}}{2}, x \geq \dfrac{2a\sqrt{3}}{2a+\sqrt{3}}; a = -\dfrac{\sqrt{3}}{2}, \text{nessun valore di } x; a > -\dfrac{\sqrt{3}}{2}, x \leq \dfrac{2a\sqrt{3}}{2a+\sqrt{3}}\right]$