

TEST YOUR SKILLS

1 TEST Let a and b be distinct real numbers for which:

$$\frac{a}{b} + \frac{a + 10b}{b + 10a} = 2.$$

Find $\frac{a}{b}$.

- A** 0.6 **B** 0.7 **C** 0.8 **D** 0.9 **E** 1

(USA American Mathematics Contest 10, 2002)

Le gare American Mathematics Contest 10 (AMC 10)

sono rivolte a studenti americani del primo biennio superiore.

2 TEST What is the value of x if $x > 0$ and $72x^2 = 9800$?

- A** $\frac{35}{3}$
B $\frac{7}{4}$
C $\frac{100}{9}$
D $3\sqrt{10}$
E $2\sqrt{30}$

(USA University of South Carolina: High School Math Contest, 2000)

3 The sum of the squares of 6 consecutive integers is 1111. What are the integers?

(USA Southeast Missouri State University: Math Field Day, 2005)

$$[11, 12, 13, 14, 15, 16; -16, -15, -14, -13, -12, -11]$$

4 The roots of the equation

$$2x^2 + 6x + 3 = 0$$

are α and β . Show that $\alpha^2 + \beta^2 = 6$.

(USA North Carolina State High School Mathematics Contest, 2003)

5 Two students attempted to solve a quadratic equation $x^2 + bx + c = 0$. Although both students did the work correctly, one miscopied the middle term and obtained the solution set $\{2, 3\}$, while the other miscopied the constant term and obtained the solution set $\{2, 5\}$. What is the correct solution set?

(USA Lehigh University: High School Math Contest, 2005)

$$\{1, 6\}$$

6 TEST $\frac{2x^2 + 3x + 1}{x^2 - 2x - 3}$ cannot represent which of the following real numbers?

- A** 3 **B** $\frac{1}{4}$ **C** $-\frac{1}{2}$ **D** -1 **E** $-\frac{1}{3}$

(USA Tennessee Mathematics Teachers Association: 39th Annual Mathematics Contest, 1995)

7 The graph of the quadratic function:

$$f: x \mapsto x^2 - 4x + 3, \quad x \in \mathbb{R},$$

cuts the axes at P , Q , and R as shown.

Find the coordinates for each of the points P , Q and R .

Write down the expression for $g(x)$ where

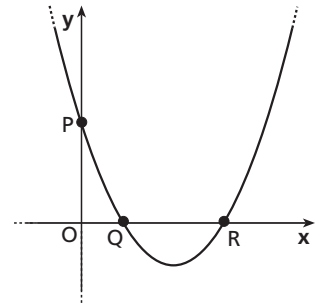
$$g(x) = f(-x).$$

Show that there are no real solutions to the equation:

$$f(x) + g(x) = 0.$$

(IR Leaving Certificate Examination, Ordinary Level, 1994)

$$[P(0; 3), Q(1; 0), R(3; 0), g(x) = x^2 + 4x + 3]$$



8 a) Factor over the rationals: $x^2 + 13x - 30$.

b) Factor over the reals: $2x^2 - 10$.

(USA Southern Illinois University Carbondale, Final Exam, 2002)

$$[a) (x + 15)(x - 2); b) 2(x - \sqrt{5})(x + \sqrt{5})]$$

9 TEST Solve for p : $\sqrt{2}p^2 - 3p + \sqrt{2} = 0$.

- A** $\{2, \sqrt{2}\}$
B $\left\{-\frac{\sqrt{2}}{2}, \sqrt{2}\right\}$
C $\left\{\sqrt{2}, \frac{\sqrt{2}}{2}\right\}$
D $\left\{-2, \frac{\sqrt{2}}{2}\right\}$
E $\left\{2, \frac{\sqrt{2}}{2}\right\}$

(USA Tennessee Mathematics Teachers Association: 39th Annual Mathematics Contest, 1995)

10 TEST How many real solutions does the equation $\frac{2}{x+2} + \frac{1}{x-3} = \frac{5}{x^2-x-6}$ have?

- A 0
- B 1
- C 2
- D 3
- E None of the above.

(USA Furman University Wylie Mathematics Tournament, 2005)

11 In what positive base b does the equation $4 \cdot 12 = 103$ for multiplication of base b numbers hold?

(USA Lehigh University: High School Math Contest, 2005)

[$b = 5$]

GLOSSARY

although: benché
to attempt: tentare
to cut-cut-cut: tagliare, intersecare
distinct: distinto, diverso
to factor: fattorizzare, scomporre
following: seguente

to hold-held-held: essere valido
integer: (numero) intero
middle: medio, centrale
quadratic: quadratico, di 2° grado
rationals: (numeri) razionali
reals: (numeri) reali

root: radice (soluzione)
to show-showed-shown: dimostrare
solution set: insieme delle soluzioni
square: quadrato