

YOU & MATHS Can you simplify this? Fully simplify $(1+x)(1+x^2)(1+x^4)(1+x^8)(1-x)$.

[USA Texas A&M University Math Contest, 2012]

Multiplication is commutative. So the product is also given by

$$(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8).$$

But $(a-b)(a+b) = a^2 - b^2$. So:

$$(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8) =$$

$$(1-x^2)(1+x^2)(1+x^4)(1+x^8) =$$

$$(1-x^4)(1+x^4)(1+x^8) = (1-x^8)(1+x^8) = 1-x^{16}.$$