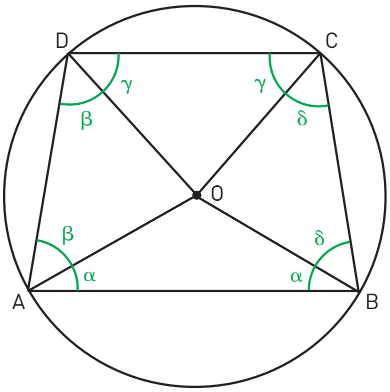
YOU & MATHS **Four in a circle** Prove that a trapezium inscribed in a circle is an isosceles one.

Let us draw a trapezium inscribed in the circle. We know that AB is parallel to CD.

We want to prove that $AD \cong BC$. Let us connect the centre of the circle with each vertex of the trapezium, drawing four line segments, OA, OB, OC, and OD. Each of these line segments is a radius of the circle, so $OA \cong OB \cong OC \cong OD$.
Thus triangles AOB, BOC, COD, and AOD are all isosceles triangles, and each of them has a pair of congruent angles: we name these angles α, β, γ, and δ, as shown in the drawing below.



We now notice that $\alpha + \beta + \gamma + \delta = 180°$, because the quadrilateral is inscribed in a circle.
Moreover $\alpha + \beta + \beta + \gamma = 180°$, because of the property of angles formed by parallel lines cut by a transversal.

Now we can write:

$$\alpha + \beta + \gamma + \delta = 180° = \alpha + \beta + \beta + \gamma \rightarrow \alpha + \beta + \gamma + \delta \cong \alpha + \beta + \beta + \gamma \rightarrow \beta \cong \delta.$$

So triangles AOD and BOC have congruent angles; since $OA \cong OB$, we get $AOD \cong BOC$ (SAS criterion of congruence), and finally we proved that $AD \cong BC$.