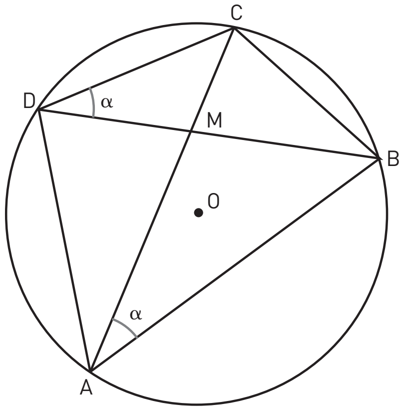
YOU & MATHS **Diagonals and angles** Let $ABCD$ be a quadrilateral. Draw its diagonals AC and BD. Prove that $ABCD$ is inscribed in a circumference if and only if $C\widehat{D}B \cong C\widehat{A}B$.

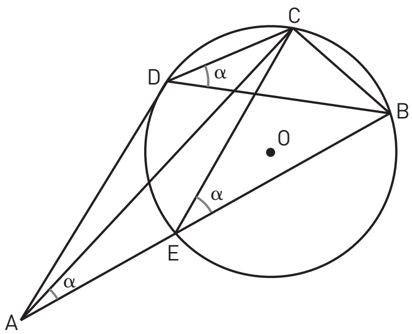
We show that if $ABCD$ is inscribed in a circle, then $C\widehat{D}B \cong C\widehat{A}B$.
We know that all angles that have their vertex on the circle and intercept the same arc are congruent.



Therefore $C\widehat{D}B \cong C\widehat{A}B$, since they intercept the same arc BC.

We now want to show that if $C\widehat{D}B \cong C\widehat{A}B$, then $ABCD$ is inscribed in a circle.

Let us draw the circle with centre O passing through the points B, C, and D.
Then let us suppose that A does not belong to the circle.
Let E be the point of intersection of AB and the circle. Then $C\widehat{D}B \cong C\widehat{E}B$, since they intercept the same arc.



Since $C\widehat{A}B \cong C\widehat{E}B$, we conclude that CA and CE are parallel, but this is impossible, since they have a point in common.

It is up to you to consider the case where A is inside the circle, which is identical to the case shown above.
We can then state that A belongs to the circle, which is therefore circumscribed around the quadrilateral.