ESERCIZI IN PIÙ
LE DISEQUAZIONI LETTERALI INTERE

Risolvi le seguenti disequazioni nell'incognita x con $a \in \mathbb{R}$.

1 $3x^2 - ax - 10a^2 > 0$

$$\left[a > 0, x < -\frac{5}{3}a \vee x > 2a; a < 0, x < 2a \vee x > -\frac{5}{3}a; a = 0, x \neq 0 \right]$$

2 $x^2 + 4a^2 + 1 + 4ax + 2x + 4a \leq 9a^2 + 12a + 4$

$$\left[a > -\frac{2}{3}, -5a - 3 \leq x \leq a + 1; a < -\frac{2}{3}, a + 1 \leq x \leq -5a - 3; a = -\frac{2}{3}, x = \frac{1}{3} \right]$$

3 $2x^3 + ax^2 - 5a^2x + 2a^3 \geq 0$

$$\left[a > 0, -2a \leq x \leq \frac{1}{2}a \vee x \geq a; a < 0, a \leq x \leq \frac{1}{2}a \vee x \geq -2a; a = 0, x \geq 0 \right]$$

4 $3x^3 - ax^2 < 12x - 4a$

$$\left[a < -6, x < \frac{a}{3} \vee -2 < x < 2; a = -6, x < 2 \wedge x \neq -2; -6 < a < 6, x < -2 \vee \frac{a}{3} < x < 2; \right.$$
$$\left. a = 6, x < -2; a > 6, x < -2 \vee 2 < x < \frac{a}{3} \right]$$

5 $(2x^2 - 13ax + 21a^2)(x^3 + 8) \geq 0$

$$\left[a < -\frac{2}{3}, \frac{7}{2}a \leq x \leq 3a \vee x \geq -2; a = -\frac{2}{3}, x \geq \frac{7}{2}a; -\frac{2}{3} < a < -\frac{4}{7}, \frac{7}{2}a \leq x \leq -2 \vee x \geq 3a; \right.$$
$$a = -\frac{4}{7}, x = -2 \vee x \geq 3a; -\frac{4}{7} < a < 0, -2 \leq x \leq \frac{7}{2}a \vee x \geq 3a; a = 0, x \geq -2;$$
$$\left. a > 0, -2 \leq x \leq 3a \vee x \geq \frac{7}{2}a \right]$$

6 $(x - 1) : \frac{12}{a - 3} + \frac{8a + x}{4} - \frac{1}{6} \geq \frac{12a - 1}{6} + \frac{5 - x}{12}$

$$\left[a < -1, x \leq \frac{a + 2}{a + 1}; a = -1, \nexists x \in \mathbb{R}; -1 < a < 3 \vee a > 3, x \geq \frac{a + 2}{a + 1}; a = 3, \text{senza significato} \right]$$

7 $\frac{ax}{3} - \frac{ax - 4x + 3a - 4}{2(a - 4)} \leq \frac{4x - a(x + 3)}{3a - 12}$

$$\left[a < \frac{1}{2}, x \geq \frac{3}{2a - 1}; a = \frac{1}{2}, \forall x \in \mathbb{R}; \frac{1}{2} < a < 4 \vee a > 4, x \leq \frac{3}{2a - 1}; a = 4, \text{senza sign.} \right]$$

8 $\left[\dfrac{a(x+3)-x}{6}-\dfrac{a-1}{2}\right]:\left(a-\dfrac{a-1}{2}+2\right)<\dfrac{2a+13}{3a+15}$

$$\left[a<-5\vee a>1,x<2\,\dfrac{a+5}{a-1};\,a=1,\forall\,x\in\mathbb{R};\,a=-5,\text{senza sign.};\,-5<a<1,x>2\,\dfrac{a+5}{a-1}\right]$$

9 $\dfrac{x}{3a}-\dfrac{1-4a^2}{1+2a}+\dfrac{1}{2a}>\dfrac{x+3(4a^2+1)-1}{6a}$

$$\left[a=-\dfrac{1}{2}\vee a=0,\text{senza sign.};\,a>0,x>6a-1;\,a<0\wedge a\neq-\dfrac{1}{2},x<6a-1\right]$$

10 $x\cdot(2a+7):(4a+10)-\dfrac{5x+3ax}{6a}>\dfrac{a+2x}{3a}-\dfrac{3}{2}\cdot\left(\dfrac{x}{a}\right)$

$$\left[a>-\dfrac{5}{2}\wedge a\neq0,x>\dfrac{2a+5}{3};\,a<-\dfrac{5}{2},x<\dfrac{2a+5}{3};\,a=-\dfrac{5}{2}\vee a=0,\text{senza sign.}\right]$$