TEST YOUR SKILLS

1 Given that $A = \{a, b, c, d, e, f\}$ and $B = \{b, d, f\}$, determine if the following statements are true or false:

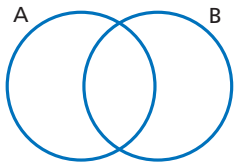
a) $B \in A$

b) $B \subset A$

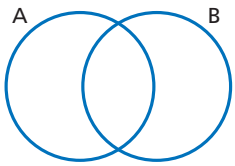
c) $B \cap A = B$

d) $B \cup A = B$

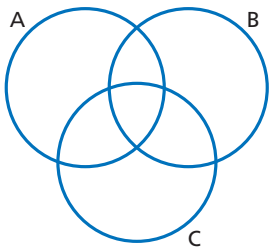
2 Shade the appropriate area for each Venn diagram.



1. $(A - B) \cup (B - A)$



2. $(A \cup \overline{B}) - \overline{A}$



3. $(A \cap \overline{C}) \cup B$

(USA *Southeast Missouri State University: Math Field Day*, 2005)

3 The set of multiples of 3 is equivalent to the set $\mathbb{N}$ of natural numbers (true/false).

(USA *Southeast Missouri State University: Math Field Day*, 2005)

4 Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{3, 4\}$, $D = \{1, 2\}$, $E = \{5, 6\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set.
Using the sets just described, describe each of the following sets by listing its elements.

a) $(\overline{B} \cup E) - A$

b) $\overline{C} \cap B$

c) $(C \cap D) - A$

d) $C \times E$

(USA *Southeast Missouri State University: Math Field Day*, 2005)

[a) $\{5, 6, 7, 8, 9\}$; b) $\{5, 6\}$; c) $\varnothing$; d) $\{(3; 5), (3; 6), (4; 5), (4; 6)\}$]

5 **TRUE OR FALSE?**

Let $A = \{1, 2, 3, 4\}$, and $B = \{3, 4, 5, 6\}$, $C = \{3, 4\}$, $D = \{1, 2\}$, $E = \{5, 6\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set.
($n(S)$ denotes the number of elements of S, «:» means «is equivalent to», $A \times B$ denotes the cartesian product.)
Using the sets just described, determine if the following statements are true or false.

a) $n(C \times E) = 8$ T F

b) $C \subset B$ T F

c) $D : E$ T F

d) $A \cup B = C \cap D \cap E$ T F

(USA *Southeast Missouri State University: Math Field Day*, 2005)

6 A certain cafeteria serves ham and cheese sandwiches, ham and tomato sandwiches, and tomato and cheese sandwiches. It is common for one meal to include multiple types of sandwiches. On a certain day, it was found that 80 customers had meals which contained both ham and cheese; 90 had meals containing both ham and tomatoes; 100 had meals containing both tomatoes and cheese. 20 customers' meals included all three ingredients. How many customers were there?

(USA *Harvard-MIT Mathematics Tournament*, 2002)

[230]

7 **TEST** Given three sets A, B and C for which the following is true. $\overline{A}$ indicates the complement of A.

a) $(A \cap \overline{B}) \cup C = \{1, 2, 3, 4, 5\}$
b) $A \cup (\overline{B} \cap C) = \{2, 3, 6, 7, 8\}$
c) $B \subset C$

If the total of the values in set A is twice the total of those in set B, what are the elements of B?

A $\{2, 3, 5\}$ D $\{1, 3, 4, 5\}$
B $\{1, 4, 5\}$ E $\{1, 2, 3, 4, 5\}$
C $\{1, 2, 4, 5\}$

(USA *North Carolina State High School Mathematics Contest*, 2004)

8 How many proper subsets does $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ have?

(USA *Southeast Missouri State University: Math Field Day*, 2005)

[2046]

9 On the Tohomo o'Odham reservation, people speak Spanish, English, and/or Tohomo o'Odham. Specifically, $\frac{3}{4}$ of the people speak Tohomo o'Odham, $\frac{1}{3}$ speak Spanish and $\frac{5}{12}$ speak English. Everyone speaks at least one of the languages. What is the largest possible fraction of the population that could speak all three languages? Use pictures to answer this question.

(USA *Arizona University*, Practice Test, 2004)

$\left[\frac{1}{4}\right]$

10 **TEST** In the village of Much-Pedling-in-the-Marsh, $\frac{1}{3}$ of the children can swim, $\frac{2}{3}$ can ride a bicycle, and $\frac{1}{7}$ can both swim and ride a bicycle (though not necessarily at the same time). Given that there are fewer than 40 children in Much-Pedling-in-the-Marsh, how many of them can neither swim nor ride a bicycle?

A 1 B 2 C 3 D 4 E 5

(UK *Senior Mathematical Challenge*, 2002)
Le gare *Senior Mathematical Challenge* sono rivolte agli studenti britannici dai 16 ai 18 anni.

TEST

11 The negation of the statement «No members of the team are male» is:

A «No members of the team are female».
B «Some members of the team are female».
C «Some members of the team are male».
D «All members of the team are male».
E «All members of the team are female».

(USA *Indiana State Mathematics Contest*, 2006)

12 Four children are arguing over a broken toy. Ali says Barbara broke it. Barbara says Tyler broke it. Tyler and Hei-Lam say they do not know who broke it. Only the guilty child was lying. The child who broke the toy was:

A Ali. B Barbara. C Tyler. D Hei-Lam. E It cannot be determined from the information given.

(USA *University of North Carolina: State Mathematics Finals Contest*, 2003)

13 The inverse of the statement «If today is Wednesday, then tomorrow is Thursday» is:

A «If today is not Wednesday, then tomorrow is not Thursday».
B «If tomorrow is Thursday, then today is Wednesday».
C «If tomorrow is not Thursday, then today is not Wednesday».
D «If today is Wednesday, then tomorrow is not Thursday».
E «If today is not Wednesday, then tomorrow is Thursday».

(USA *Northern State University: 48th Annual Mathematics Contest*, 2001)

14 The contrapositive of the statement «If today is Wednesday, then tomorrow is Thursday» is:

A «If today is not Wednesday, then tomorrow is not Thursday».
B «If tomorrow is Thursday, then today is Wednesday».
C «If tomorrow is not Thursday, then today is not Wednesday».
D «If today is Wednesday, then tomorrow is not Thursday».
E «If today is not Wednesday, then tomorrow is Thursday».

(USA *Northern State University: 49th Annual Mathematics Contest*, 2002)

GLOSSARY

to argue: discutere
at least: almeno
both: entrambi
to break-broke-broken: rompere
certain: dato, certo
cheese: formaggio
complement: complemento
contrapositive: contronominale
customer: cliente
equivalent: equivalente (equipotente)
female: femmina
fewer than: meno di
guilty: colpevole
ham: prosciutto
inverse: inverso
to lie-lied-lied (lying): mentire
to list: elencare
male: maschio
meal: pasto
multiple: multiplo
negation: negazione
picture: disegno
proper (subset): (sottoinsieme) proprio
reservation: riserva
to ride-rode-ridden: montare, andare in
set: insieme
to shade: ombreggiare
statement: enunciato, frase, proposizione
to swim-swam-swum: nuotare
team: squadra
though: benché, anche se
tomato: pomodoro
toy: giocattolo
universal set: insieme universo