🇬🇧 TEST YOUR SKILLS

1 ⬤⬤⬤ **TEST** Let $f: \mathbb{N} \to \mathbb{N}$ be a function such that $f(1) = 1$ and $3f(n + 1) - 3f(n) = 1$ for all $n \geq 1$. Find $f(2002)$.

$$\left(\text{Hint:} f(n + 1) = f(n) + \frac{1}{3} = f(n - 1) + \frac{2}{3} = \dots\right)$$

 A 666   **B** 667   **C** 668   **D** 669   **E** 670

(USA *American Mathematics Contest* 10, 2002)
Le gare *American Mathematics Contest 10* (*AMC 10*)
sono rivolte a studenti americani del primo biennio superiore.

2 Let f and g be two functions defined on $\mathbb{R}$ by:

$$f: x \mapsto 2x - 1, \qquad g: x \mapsto \frac{7x - 2}{3}.$$

 a) Establish if $f(3a + 2b) = 3f(a) + 2f(b)$, for all $a, b \in \mathbb{R}$.

 b) Express $g(a + b)$ in the form $g(a) + g(b) + c$, $c \in \mathbb{R}$.

 c) Plot the graphs of the two functions and hence estimate the value of x for which $f(x) = g(x)$. $\left[\text{a) no; b) } c = \frac{2}{3}; \text{c) } x = -1\right]$

3 If $f(x) = \dfrac{3}{1 + x}$ for all $x \neq -1$, then

$3f(x) = \dots$

(USA *Lehigh University: High School Math Contest*, 2005)

4 **TEST** Tables for f and g are given. Use them to evaluate $(g \circ f)(-2)$ if possible.

x	$f(x)$
-2	1
0	4
2	3
4	2

x	$g(x)$
1	2
2	4
3	-2
4	0

 A -2 **B** 0   **C** 2   **D** 4   **E** Undefined.

(USA *University of Wyoming, Test*, 2004)

5 Let $f(x) = x^2 + 3$ and $g(x) = 2x - 1$. Find and expand completely $f(g(x))$.

(USA *Southern Illinois University Carbondale, Final Exam*, 2003)

$$[f(g(x)) = 4x^2 - 4x + 4]$$

6 **TEST** Which of the following is true of the relation h whose domain and range are both over the set of all integers such that
$h(n) = 4n - 1$?
I. It is a function.
II. It is one-to-one.
III. It is onto.
IV. It decreases as the value of n increases.
 A I only.
 B All are true.
 C I and II.
 D I and III.
 E None.

(USA *Vanderbilt High School Mathematics Competition*, 2006)

GLOSSARY

domain: dominio	**graph**: grafico
to establish: stabilire	**hence**: quindi, inoltre
to estimate: stimare	**one-to-one**: iniettiva
to evaluate: valutare, calcolare	**onto**: suriettiva
to expand: espandere, sviluppare	**range**: codominio
	relation: relazione
	table: tabella