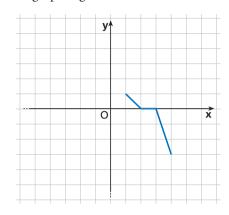
TEST YOUR SKILLS

Part of a graph is given below.



Sketch the rest of this graph if the graph has the indicate symmetry.

- a) y-axis symmetry.
- b) *x*-axis symmetry.
- c) Origin symmetry.
- d) Both *y*-axis and origin symmetry.

(USA Tacoma Community College, Worksheet)

- **2 TEST** Which of the following functions is symmetric respect to the *y*-axis?
 - **A** y = 2x
 - **B** $y = x^2 + 2$
 - **C** $y = (x 2)^2$
 - **D** $y = (x + 2)^2$
 - **E** $y = x^2 + 2x$

(USA University of Wyoming, Practice Test, Finals)

3 Determine (algebraically) if each function is even, odd, or neither.

a) y = |x + 2|. b) y = |x| + 2.

- y = |x| + 2
- c) $y = x^2 + 3$.
- $d) y = x^2 + 3x.$
- e) $y = x^3 5x$. f) $y = x^3 - 5$.

(USA *Tacoma Community College*, Worksheet) [a) neither; b) even; c) even; d) neither; e) odd; f) neither] **4 TEST** Starting with the point (3; 4) in the cartesian plane, reflect it across the *x*-axis, then rotate it 180° around the origin, and then finally translate it vertically by 5 units. The final point is:

A
$$(8; -4).$$
 D $(-4; 8).$

 B $(3; 9).$
 E $(-3; 9).$

 C $(-3; 1).$

(USA Marywood University Mathematics Contest, 2001)

5 TEST The point (a; b) is reflected over the *y*-axis to the point (c; d), which is reflected over the *x*-axis to the point (e; f). Compute ab - ef.

- - 2. L
- **C** 2*ab*.

(USA University of North Carolina: State Mathematics Finals Contest, 2003)

6 TEST Let *P* be the point (3; 2). Let *Q* be the reflection of *P* about the *x*-axis, let *R* be the reflection of *Q* about the line y = -x and let *S* be the reflection of *R* through the origin. Then *PQRS* is a convex quadrilateral. What is the area of *PQRS*? **A** 14 **B** 15 **C** 16 **D** 17 **E** 18

(USA North Carolina: State High School Mathematics Contest, 1997)

7 Suppose that a particle sitting at the point (3; 6) is rotated 30° in the clockwise direction about the point (-1; 2). Give the coordinates of the new location of the particle.

(USA Texas A&M University Math Contest, 1999) $[(2\sqrt{3}+1; 2\sqrt{3})]$

8 Find the vertex of the parabola $x^2 + 2y = 2$. (USA Southern Illinois University Carbondale, Final Exam, 2002) [(0; 1)]

TEST

9 Given that the vertex of the parabola $y = x^2 + 8x + k$ is on the x-axis, what is the value of k? A 0 B 4 C 8 D 16 E 24

(USA University of South Carolina: High School Math Contest, 2001)

10 The graph of two parabolas $y = 2x^2$ and $y = x^2 + x + 6$ intersect in two points. An equation for the line that passes through these two points is:

A
$$x - 2y + 18 = 0$$
.
b $2x - y + 4 = 0$.
c $2x - y - 18 = 0$.
c $2x - y + 12 = 0$.

(USA North Carolina State High School Mathematics Contest, 2003)

- 11 Let P(a; b) and Q(c; d) denote two distinct points on the graph of $y = x^2$. Suppose that the slope of line *PQ* is 5 and the *x* coordinates of *P* and *Q* differ by 1. Find b + d.
 - **A** 41.
 - **B** 25. **E** None of these.
 - **C** 13.

(USA North Carolina State High School Mathematics Contest, 2003)

D 5.

12 A toy rocket is fired vertically from the ground. Its height in meters above the ground is given by $s(t) = 36t - 4.9t^2$, where *t* represents the time in seconds. What is the maximum height of the rocket?

(USA Southeast Missouri State University: Math Field Day, 2005) [66.12 m]

13 Let
$$y = -4x^2 - 2x + 3$$
.

- a) Find the *y*-intercept. Find the vertex and axis of symmetry. Sketch.
- b) Find the *x*-intercept, if any. Indicate them on your graph of part *a*.
- c) Determine whether *y* has a maximum value or a minimum value and state that value.

(USA Southern Illinois University Carbondale, Final Exam, 2002)

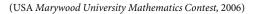
$$\begin{bmatrix} a \end{pmatrix} y = 3; V\left(-\frac{1}{4}; \frac{13}{4}\right); x = -\frac{1}{4};$$

b) $x = \frac{-1 \pm \sqrt{13}}{4}; c) \max y = \frac{13}{4}$

TEST

The maximum value of the quadratic function $y = -x^2 + 4x - 1$ is:

- **A** 3.
- **B** 3.
- C 2.
- **D** 5.
- **E** 5.



- 15 Which of the following statements regarding $y = 3x^2 12$ are true?
 - 1. The slope is 3.
 - 2. *y* is increasing for all *x*.
 - 3. The *x*-intercepts are $(\pm 2; 0)$.
 - 4. The *y*-intercept is (0; -12).
 - 5. The vertex is (0; -12).
 - **A** 1, 2, and 3.
 - **B** 3, 4, and 5.
 - **C** 3 and 4.
 - **D** 2 and 4.
 - **E** 3 and 5.

(USA University of Wyoming, Test, 2004)

6 The vertex of the graph of the quadratic function $y = 2x^2 + 4x - 3$ is at the point:

- **▲** (0; − 3).
- **B** (2; 13).
- **C** (−1; − 5).
- **D** (1; 3).
- [E] (-1; 5).

(USA Marywood University Mathematics Contest, 2001)

- **17** Given a circle centered at (3; 4) that passes through point (7; 1), which of the following is the equation of the tangent line to the circle at point (7; 1)?
 - $\boxed{A} 4x 3y = 25$
 - **B** 3x + 4y = 25
 - **C** 3x 4y = 17
 - **D** 4x + 3y = 31
 - **E** None of these.

(USA North Carolina State High School Mathematics Contest, 2004)

- 18 What is the *y*-component of the center of the circle which passes through (-1; 2), (3; 2) and (5; 4)? (USA Lehigh University: High School Math Contest, 2001)
- **19** Find the equation of the circle that has a diameter with endpoints (1; 1) and (7; 5).

(USA Southern Illinois University Carbondale, Final Exam, 2003) $[x^{2} + y^{2} - 8x - 6y + 12 = 0]$

Find the radius of a circle whose equations is $x^2 + y^2 = 12y - 10x - 10$.

(USA Indiana State Mathematics Contest, 2006) $\sqrt{51}$

TEST

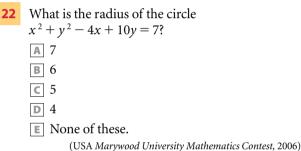
21 An ellipse is centered at (0; 0) with foci at (-4; 0). Its eccentricity e = 0.8. What is the equation for the ellipse?

A
$$\frac{x^2}{64} + \frac{y^2}{16} = 1$$

B $\frac{x^2}{25} + \frac{y^2}{16} = 1$
C $\frac{x^2}{25} + \frac{y^2}{9} = 1$
D $\frac{x^2}{16} + \frac{y^2}{9} = 1$

E None of these.

(USA Marywood University Mathematics Contest, 2006)



- **23** Circle *U* passes through points (3; 11), (11; -1) and (-14; 4). Find the coordinates of the center of circle *U*.
 - **A** (-2; -1).
 - **B** (-1; -2).
 - **C** (2; 1).
 - **D** (1; 2).
 - **E** None of the above.
 - (USA Catawba College NCCTM Mathematics Contest, 2005)

24 Classify the curve $\frac{x^2}{4} + \frac{y^2}{36} = 1$ as one of parabola, ellipse, or hyperbola. Sketch the graph, with foci and vertices clearly labeled. [ellipse]

25 Another way to define eccentricity for a hyperbola or an ellipse is $e = \frac{c}{c}$. Find the eccentricity of

the ellipse
$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

(USA Central Lake College, Worksheet)

 $\left[e = \frac{3}{5}\right]$

GLOSSARY

algebraically:	endpoint: estremo	line: retta	slope: pendenza
algebricamente	even: pari	odd: dispari	statement: affermazione
center: centro	to fire: sparare	quadratic (function):	tangent line: retta
circle: circonferenza	focus (foci): fuoco (fuochi)	(funzione) quadratica	tangente
clockwise: in senso orario	graph: grafico	radius: raggio	toy rocket: razzo
to denote: denotare	ground: suolo	to reflect: riflettere	giocattolo
eccentricity: eccentricità	height: altezza	to regard: riguardare	to translate: traslare
eccentricity: eccentricita	height: altezza	to regard: riguardare	to translate: traslare
ellipse: ellisse	hyperbola: iperbole	to rotate: ruotare	vertex : vertice