YOU & MATHS Factor completely:

a. $x^2 - 11xy + 30y^2$

b. $4x^5 - 12x^4 - 40x^3$

c. $x^2 - 2x + 5x - 10$

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a. Let's look for a and b such that:

$$x^2 - 11xy + 30y^2 = (x + ay) \cdot (x + by) = x^2 + (a + b)xy + aby^2.$$

We thus have to solve:

$$\begin{cases} a + b = -11 \\ a \cdot b = 30 \end{cases}$$

Let's consider all the possible integer values of a, b such that $a \cdot b = 30$:

$$\pm 1, \pm 30; \pm 2, \pm 15; \pm 3, \pm 10; \pm 5, \pm 6.$$

The only pair that satisfies the condition $a + b = -11$ is $a = -5$, $b = -6$.
Now we can factor the initial polynomial:

$$(x - 6y) \cdot (x - 5y).$$

b. We can start by factoring out $4x^3$:

$$4x^5 - 12x^4 - 40x^3 = 4x^3(x^2 - 3x - 10).$$

Now we have to find the zeros of $A(x) = x^2 - 3x - 10$ among the divisors of its constant term 10, so let's see if any value among $\pm 1, \pm 2, \pm 5, \pm 10$ is a zero of the poynomial:

$$A(+1) = 1 - 3 - 10 \neq 0;$$
$$A(-1) = 1 + 3 - 10 \neq 0;$$
$$A(+2) = 4 - 6 - 10 \neq 0;$$
$$A(-2) = 4 + 6 - 10 \neq 0.$$

We conclude that -2 is a zero of the polynomial.
To find the other zero of $A(x) = x^2 - 3x - 10$ we can try the remaining divisors, use the Ruffini rule, or notice that the product of its two zeros must be -10. Using that trick, we conclude that the other zero is 5.
Finally, we factor the initial polynomial as:

$$4x^3 \cdot (x + 2) \cdot (x - 5).$$

c. Thanks to how the polynomial is written, we notice that:

$$x^2 + 5x - 2x - 10 =$$
$$x^2 - 2x + 5x - 10 =$$
$$x(x - 2) + 5(x - 2) =$$
$$(x - 2) \cdot (x + 5).$$