

**YOU & MATHS** Roger enjoys soccer and his team is pretty good: last season they won 7 more games than they lost. There were a total of 23 games; none were ties. How many games did his team win?

(CAN John Abbott College, Final Exam, 1999)

Let  $x$  be the number of games that Roger's team won in the last season and  $y$  the number of games that they lost. The problem tells us that:

- they won 7 more games than they lost  $\rightarrow x = y + 7$ ;
- there were a total of 23 games and they did not score any ties (i.e. they always won or lost)  $\rightarrow x + y = 23$ .

Using Cramer's rule, we want to solve the linear system:

$$\begin{cases} x = y + 7 \\ x + y = 23 \end{cases}$$

We first rearrange the first equation, in order to put the system in standard form:

$$\begin{cases} x - y = 7 \\ x + y = 23 \end{cases}$$

The determinant  $D$  of the system is:

$$D = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 \cdot 1 - 1 \cdot (-1) = 1 + 1 = 2$$

and, as it is different than zero, we know that we can find two real solutions for  $x$  and  $y$ .

Let us calculate  $D_x$  by substituting the constant terms to the first column of coefficients of  $x$  in the above matrix, and  $D_y$  by substituting the constant terms to the second column of coefficients of  $y$ . Then:

$$D_x = \begin{vmatrix} 7 & -1 \\ 23 & 1 \end{vmatrix} = 7 \cdot 1 - 23 \cdot (-1) = 7 + 23 = 30;$$

$$D_y = \begin{vmatrix} 1 & 7 \\ 1 & 23 \end{vmatrix} = 1 \cdot 23 - 1 \cdot 7 = 23 - 7 = 16.$$

We use the following formulae to calculate  $x$  and  $y$ :

$$x = \frac{D_x}{D} = \frac{30}{2} = 15;$$

$$y = \frac{D_y}{D} = \frac{16}{2} = 8.$$

Roger's team won 15 games in the last season.