YOU & MATHS How many different independent isometries transform a regular pentagon in itself?

Let us recall that the isometries we have studied are: translation, rotation and reflection. Let us examine them case by case.

a. There are no translations which transform whatever shape in itself. (We are not considering a translation by a null vector because it does not do anything to the shape.)

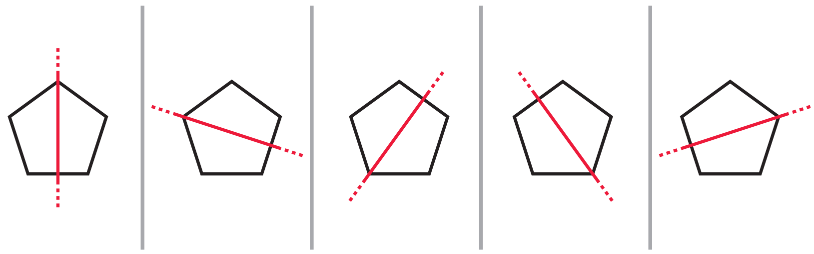
b. If we rotate a regular pentagon by an angle of $\frac{360°}{5} = 72°$, the pentagon will be transformed into itself. Re-applying the same rotation to the rotated pentagon is equivalent to rotating the initial pentagon by an angle of $2 \cdot \frac{360°}{5} = 144°$. Therefore we have 5 independent rotations which transform a regular pentagon in itself:

$$n \cdot \frac{360°}{5} \qquad (n = 1, 2, 3, 4, 5).$$

For $n = 5$ we have a 360° rotation, which is an identity.

Note that the direction of the rotation is irrelevant because a $n \cdot \frac{360°}{5}$ clockwise rotation is equivalent to a $(5 - n) \cdot \frac{360°}{5}$ counterclockwise rotation.

c. There are no points in a regular pentagon that can be used as a centre of symmetry. Let us therefore consider only the reflection across a line. The regular pentagon can be reflected across the 5 following axes of symmetry.



We can therefore conclude that there are different independent isometries that map a regular pentagon onto itself.