

YOU & MATHS Which of the following numbers is a perfect square?

- A** $4^4 5^5 6^6$ **B** $4^4 5^6 6^5$ **C** $4^5 5^4 6^6$ **D** $4^6 5^4 6^5$ **E** $4^6 5^5 6^4$

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sono rivolte a ragazzi americani del secondo biennio superiore.

A perfect square is a natural number whose square root is a natural number. For instance, 9 and 25 are perfect squares of 3 and 5. If we were to write this in power form, we would get:

$$9 = 3 \cdot 3 = 3^2,$$

$$25 = 5 \cdot 5 = 5^2.$$

Let's also have a quick look at the following examples:

$$625 = 25 \cdot 25 = 5 \cdot 5 \cdot 5 \cdot 5 = 5^4,$$

$$324 = 18 \cdot 18 = 2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^4.$$

We notice that all of the factors that make up a perfect square have exponents that are even numbers.

That should help us choose our answer, but if we look at the five choices we have, none of them has all even exponents. We notice, though, that the factor 4 is equal to $2 \cdot 2$, so all its powers can be rewritten as powers of 2. For each of the possible answers, we get:

$$4^4 = 4 \cdot 4 \cdot 4 \cdot 4 = 2 \cdot 2 = 2^8,$$

$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 2 \cdot 2 = 2^{10},$$

$$4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 2 \cdot 2 = 2^{12}.$$

If we substitute the new powers into the five possible answers, we notice that one of them has all even exponents. It is:

$$4^5 5^4 6^6 = 2^{10} 5^4 6^6,$$

that is, answer C.