

YOU & MATHS Harvesting corn For the past few years Old Happy Farm had harvested about 1000 more corncobs than Billy's Farm. However this year, both farms' harvests were cut down by one fourth. To compensate for the loss, the rival farms bought equal amounts of corn from neighboring States. Which farm ended up having more corn? Or did they have equal amounts?

You can call H the number of corncobs at Old Happy Farm, and B the number of corncobs at Billy's Farm. Then in a normal year $H = B + 1000$, so $H > B$. Therefore, this year the corn production satisfies the inequality $\frac{3}{4}H > \frac{3}{4}B$ because the direction of the inequality stays the same for multiplication by positive numbers. If c is the number of corncobs bought by both the farms from neighboring States, the inequality becomes

$$\frac{3}{4}H + c > \frac{3}{4}B + c.$$

If we substitute H with $B + 1000$, we get

$$\frac{3}{4}(B + 1000) + c > \frac{3}{4}B + c, \text{ which is } \frac{3}{4}B + 750 + c > \frac{3}{4}B + c.$$

Now you can reduce the $\frac{3}{4}B$ from both sides and end up with $750 + c$ on one side and c on the other. The inequality is still true, because 750 plus a quantity is greater than the quantity alone, so Old Happy Farm still had more corn.