

YOU & MATHS Find the sum of the integer numbers from -50 to $+52$, including -50 and $+52$.

(USA Bay Area Math Meet, Bowl Sampler, 1997)

This exercise seems to imply a long string of calculations, but with some insight we can solve it quickly.

The sum that we have to calculate is

$$(-50) + (-49) + (-48) + \dots + (-1) + 0 + (+1) + \dots + (+48) + (+49) + (+50) + (+51) + (+52).$$

Looking closely, we notice that many of the positive numbers have corresponding negative numbers that are their opposite. For instance, -50 and $+50$, -49 and $+49$, and so on. Let's order these:

$$(-50) + (+50) + (-49) + (+49) + (-48) + (+48) + \dots + (-1) + (+1) + (+51) + (+52).$$

These pairs of numbers add up to zero, as follows:

$$(-50) + (+50) = 0,$$

$$(-49) + (+49) = 0,$$

$$(-48) + (+48) = 0,$$

...

$$(-1) + (+1) = 0,$$

so if we rewrite the sum, we're left only with:

$$\cancel{(-50)} + \cancel{(+50)} + \cancel{(-49)} + \cancel{(+49)} + \cancel{(-48)} + \cancel{(+48)} + \dots + \cancel{(-1)} + \cancel{(+1)} + (+51) + (+52) = (+51) + (+52),$$

which gives us 103 as our final result.