YOU & MATHS Julia enjoys making up maths riddles to ask her friends. One day she comes up with this one: «I'm thinking of two integer numbers whose sum is −21 and whose product is 108. What numbers am I thinking of?» How would you answer Julia?

It was easy for Julia to make up this riddle, because all she had to do was think of two numbers and give us their sum and product. But it's going to be a little more difficult for us to solve it.
First, let's call p the product of the two numbers, 108, and s their sum, −21. Now let's take all the integers whose product is 108 and put them in a table.

P
$(+2)(+54) = 108$
$(-2)(-54) = 108$
$(+4)(+27) = 108$
$(-4)(-27) = 108$
$(+12)(+9) = 108$
$(-12)(-9) = 108$
$(+36)(+3) = 108$
$(-36)(-3) = 108$
$(+108)(+1) = 108$
$(-108)(-1) = 108$

Next, we can calculate the sum of each pair of numbers and write it in a new column.

p	*s*
$(+2)(+54) = 108$	$+2+54=+56$
$(-2)(-54) = 108$	$-2-54=-56$
$(+4)(+27) = 108$	$+4+27=+31$
$(-4)(-27) = 108$	$-4-27=-31$
$(+12)(+9) = 108$	$+12+9=+21$
$(-12)(-9) = 108$	$-12-9=-21$
$(+36)(+3) = 108$	$+36+3=+39$
$(-36)(-3) = 108$	$-36-3=-39$
$(+108)(+1) = 108$	$+108+1=+109$
$(-108)(-1) = 108$	$-108-1=-109$

We can see that the two numbers Julia has in mind are −9 and −12.