

YOU & MATHS An operation « Δ » is defined by $a \Delta b = 1 - \frac{a}{b}$, $b \neq 0$. What is the value of $(1 \Delta 2) \Delta (3 \Delta 4)$?

(CAN Canadian Open Mathematics Challenge, 2000)

Let's apply the definition of the operation Δ to the first terms between parentheses:

$$(1 \Delta 2) = 1 - \frac{1}{2} = \frac{1}{2}.$$

Similarly, for $3 \Delta 4$ we obtain:

$$(3 \Delta 4) = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}.$$

So far we have shown that:

$$(1 \Delta 2) \Delta (3 \Delta 4) = \frac{1}{2} \Delta \frac{1}{4}.$$

Now we can use the definition of the operation Δ to get our result:

$$\frac{1}{2} \Delta \frac{1}{4} = 1 - \frac{\frac{1}{2}}{\frac{1}{4}} = 1 - \frac{1}{2} : \frac{1}{4} = 1 - \frac{1}{\cancel{2}_1} \cdot \cancel{4}^2 = 1 - 2 = -1.$$