**YOU & MATHS** Mr. Day's advisees consist of 16 students enrolled in algebra, 20 in geometry, and 12 in history. Six of this group are in both algebra and geometry, 4 are in both geometry and history, 5 are in both algebra and history, and 3 are enrolled in all three courses. Assuming that each of his advisees is enrolled in at least one of three classes, determine the number of Mr. Day's advisees.

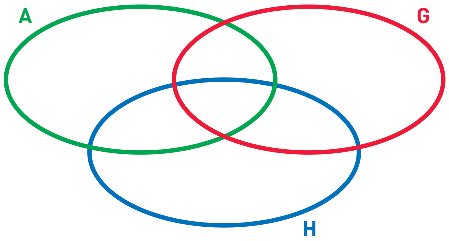
(USA *Southeast Missouri State University: Math Field Day*, 2005)

First of all, let's identify the sets the problem is talking about and draw their diagram.

$A = \{\text{students enrolled in algebra}\}$ → 16 students;

$G = \{\text{students enrolled in geometry}\}$ → 20 students;

$H = \{\text{students enrolled in history}\}$ → 12 students.




We can calculate the total number of advisees Mr. Day has by adding:

- the number of students enrolled in all three courses;
- the number of students enrolled only in two courses;
- the number of students enrolled only in one course.

This way we can be sure not to count the same student twice.

The problem already gives us the first number. Using set notation:

$A \cap G \cap H = \{\text{students enrolled in all three courses}\}$ → 3 students.

Using the data from the problem, we can define the students enrolled in two courses (or more) as follows:

$A \cap G = \{\text{students enrolled in both algebra and geometry}\}$ → 6 students;

$G \cap H = \{\text{students enrolled in both geometry and history}\}$ → 4 students;

$A \cap H = \{\text{students enrolled in both algebra and history}\}$ → 5 students.

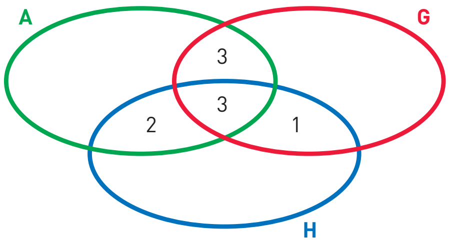
We notice that $A \cap G \cap H$ is a subset of all these three sets, so to find out the number of students enrolled only in two courses, we have to consider the following differences:

$(A \cap G) - (A \cap G \cap H) = \{\text{students enrolled only in algebra and geometry}\}$ → $6 - 3 = 3$ students;

$(G \cap H) - (A \cap G \cap H) = \{\text{students enrolled only in geometry and history}\}$ → $4 - 3 = 1$ students;

$(A \cap H) - (A \cap G \cap H) = \{\text{students enrolled only in algebra and history}\}$ → $5 - 3 = 2$ students.

Let's put these numbers in the diagram sketched earlier.



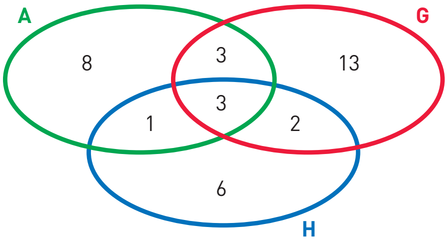
Now let's work on the number of students enrolled <u>only</u> in one course. Recalling that $A$ has 16 elements, $G$ has 20 and $H$ has 12, we can easily find out that:

no. of students only taking algebra $= 16 - 3 - 3 - 2 = 8$;

no. of students only taking geometry $= 20 - 3 - 3 - 1 = 13$;

no. of students only taking history $= 12 - 3 - 2 - 1 = 6$.

Finally, let's update our diagram with these numbers.




Therefore, the total number of students considered in this problem, Mr. Day's advisees, is given by:

$3 + 3 + 2 + 1 + 8 + 13 + 6 = 36$.