

YOU & MATHS Find $f(f(f(f(\dots f(10)\dots))))$, where the above expression contains 97 f s and $f(x) = \frac{5}{x}$.

(USA Bay Area Math Meet, BAMM, Bowl Sampler, 1997)

First of all, let's apply the definition of f for $x = 10$:

$$f(10) = \frac{5}{10} = \frac{1}{2},$$

then apply the definition for $x = \frac{1}{2}$:

$$f\left(\frac{1}{2}\right) = \frac{5}{\frac{1}{2}} = 2 \cdot 5 = 10.$$

If we sequentially apply f , we obtain:

$$f(10) = \frac{1}{2},$$

$$f(f(10)) = f\left(\frac{1}{2}\right) = 10,$$

$$f(f(f(10))) = f\left(f\left(\frac{1}{2}\right)\right) = f(10) = \frac{1}{2}, \dots$$

We deduce that:

- if we apply f an **odd** number of times, the result is $\frac{1}{2}$;
- if we apply f an **even** number of times, the result is 10.

Therefore, since 97 is an odd number, the result is:

$$f(f(f(\dots f(10)\dots))) = \frac{1}{2}.$$