

YOU & MATHS A function $f(x)$ has the following properties:

- a.** $f(1) = 1$;
- b.** $f(2x) = 4f(x) + 6$;
- c.** $f(x + 2) = f(x) + 12x + 12$.

Calculate $f(6)$.

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We notice that $6 = 4 + 2$, so we can calculate $f(6)$ by first calculating $f(2)$ and $f(4)$.

We notice that $2 = 2 \cdot 1$, so we can calculate $f(2)$ using properties **a** and **b** and $x = 1$:

$$f(2) = f(2 \cdot 1) = 4 \cdot f(1) + 6 = 4 \cdot 1 + 6 = 10.$$

Using again property **b** and $4 = 2 \cdot 2$, we can calculate $f(4)$:

$$f(4) = f(2 \cdot 2) = 4 \cdot f(2) + 6 = 40 + 6 = 46.$$

With property **c** and $6 = 4 + 2$, we can now find $f(6)$:

$$f(6) = f(4 + 2) = f(4) + 12 \cdot 4 + 12 = 46 + 60 = 106.$$