

YOU & MATHS Jeff, Gareth and Ina all share the same birthday. Gareth is one year older than Jeff, and Ina is two years older than Gareth. This year the sum of their ages is 118. How old is Gareth?

(CAN Canadian Open Mathematics Challenge, 2003)

Let x be Jeff's age, y be Gareth's age and z be Ina's age. From the problem we know that they were all born on the same day, so their birthdate differs only in the year (and not in the day or month). We also know that:

- Gareth is one year older than Jeff $\rightarrow y = x + 1$;
- Ina is two years older than Gareth $\rightarrow z = y + 2$;
- the sum of their ages is 118 $\rightarrow x + y + z = 118$.

In order to find Gareth's age (i.e. y), we need to solve the linear system:

$$\begin{cases} y = x + 1 \\ z = y + 2 \\ x + y + z = 118 \end{cases}.$$

We choose to solve it using the method of substitution. If we plug the first equation into the second and the third one, we get:

$$\begin{cases} y = x + 1 \\ z = x + 1 + 2 \\ x + x + 1 + z = 118 \end{cases} \rightarrow \begin{cases} y = x + 1 \\ z = x + 3 \\ 2x + z = 117 \end{cases},$$

and then we substitute the second equation into the third one:

$$\begin{cases} y = x + 1 \\ z = x + 3 \\ 2x + x + 3 = 117 \end{cases} \rightarrow \begin{cases} y = x + 1 \\ z = x + 3 \\ 3x = 114 \end{cases} \rightarrow \begin{cases} y = x + 1 \\ z = x + 3 \\ x = 38 \end{cases}.$$

Finally, substituting the result we have found for x into the other two equations, we have:

$$\begin{cases} y = 38 + 1 = 39 \\ z = 38 + 3 = 41 \\ x = 38 \end{cases}.$$

Then Gareth is 39 years old.