YOU & MATHS **Root free numbers** Which of the following numbers can you write without using the square root symbol $\sqrt{\ }$?

$\sqrt{10 \cdot 4000}$; $\sqrt{\dfrac{800}{2^3 \cdot 5^2}}$; $\sqrt{9 \cdot 10^9}$; $\sqrt{0,001}$; $\sqrt{\dfrac{160}{0,01}}$.

We can write the following numbers without using the root symbol:

$$\sqrt{10 \cdot 4000} = \sqrt{4 \cdot 10^4} = \sqrt{40\,000} = 200,$$

$$\sqrt{\frac{800}{2^3 \cdot 5^2}} = \sqrt{\frac{800}{200}} = \sqrt{4} = 2.$$

We cannot write the other given numbers without using the root symbol.
In fact we can simplify the radical and get rid of the root symbol if the radicand is a rational number raised to the second power.

Let us consider $\sqrt{0,001}$; we can write

$$\sqrt{0,001} = \sqrt{\frac{1}{1000}},$$

but, since 1000 is not a square number, there is no rational number $q \in \mathbb{Q}$ such that $q^2 = 0,001$, and so we cannot simplify the radical.

Then let us consider $\sqrt{9 \cdot 10^9}$; we can write:

$$\sqrt{9 \cdot 10^9} = \sqrt{9 \cdot 10^8 \cdot 10}.$$

The radicand is an integer number which is the product of three factors: two of them, 9 and 10^8, are square numbers, but the third factor, 10, is not a square number. So the whole radicand is not a square number, and we cannot simplify the radical.

Finally let us consider

$$\sqrt{\frac{160}{0,01}} = \sqrt{\frac{160}{\frac{1}{100}}} = \sqrt{16\,000} = \sqrt{16 \cdot 100 \cdot 10}.$$

The radicand is an integer number which is the product of three factors: two of them, 16 and 100, are square numbers, but the third factor, 10, is not a square number. So the whole radicand is not a square number, and we cannot simplify the radical.