YOU & MATHS For how many integer values of n does the equation $x^2 + nx - 16 = 0$ have integer solutions?

(**HINT** Remember the relation between the solutions and the coefficients of the equation.)

- **A** 2
- **B** 3
- **C** 4
- **D** 5
- **E** 6

(UK *Senior Mathematical Challenge*, 2002)

The relation between the solutions and the coefficients of an equation allows us to say that, if we consider the quadratic equation $x^2 + sx + p = 0$, then s is equal to the sum of the two solutions x_1 and x_2 of the equation, while p is equal to their product.

In this problem, the equation is $x^2 + nx - 16 = 0$ and $p = x_1x_2 = -16$. The only possible integer values for the two solutions so that their product is -16 are:

$\{-16, 1\}, \{-8, 2\}, \{-4, 4\}, \{-2, 8\}, \{-1, 16\}$.

We notice that $n = -s$, so for each of the couples of solutions above the value of n is:

$n = -(-16 + 1) = 15,$

$n = -(-8 + 2) = 6,$

$n = -(-4 + 4) = 0,$

$n = -(-2 + 8) = -6,$

$n = -(-1 + 16) = -15.$

We have therefore proved that there are only 5 integer values of n for which our equation has integer solutions.