YOU & MATHS **Absolute values** The absolute value of the difference between two numbers is 6. The sum of the absolute values of the two numbers is 14. Find the possible pairs of numbers.

The equalities that define the two numbers x and y are:

$$\begin{cases} |x-y| = 6 \\ |x|+|y| = 14 \end{cases}.$$

To avoid confusion, let us consider all possible cases in order.

a. $0 \le y \le x$. Then the equalities are: $\begin{cases} x-y=6 \\ x+y=14 \end{cases}$.

b. $y \le x \le 0$. Then the equalities are: $\begin{cases} x-y=6 \\ -x-y=14 \end{cases}$.

c. $y \le 0 \le x$. Then the equalities are: $\begin{cases} x-y=6 \\ x-y=14 \end{cases}$.

Note that since $|x-y| = |y-x|$ there is no need to consider other combinations.
For example, the case $0 \le x \le y$ has the same solutions of the case $0 \le y \le x$, as long as you exchange the role of x and y. The same applies to the remaining cases $x \le y \le 0$ (it has the same solutions of case **b**) and $x \le 0 \le y$ (it has the same solutions of case **c**).
Let us then consider the three following cases.

a. $$\begin{cases} 0 \le y \le x \\ x-y=6 \\ x+y=14 \end{cases} \to \begin{cases} 0 \le y \le x \\ x=y+6 \\ y+6+y=14 \end{cases} \to \begin{cases} 0 \le y \le x \\ x=y+6 \\ 2y=8 \end{cases} \to \begin{cases} 0 \le y \le x \\ x=10 \\ y=4 \end{cases}$$

The solution $x=10$ and $y=4$ is acceptable because $0 \le 4 \le 10$.

b. $$\begin{cases} y \le x \le 0 \\ x-y=6 \\ -x-y=14 \end{cases} \to \begin{cases} y \le x \le 0 \\ x=y+6 \\ -y-6-y=14 \end{cases} \to \begin{cases} y \le x \le 0 \\ x=y+6 \\ -2y=20 \end{cases} \to \begin{cases} y \le x \le 0 \\ x=-4 \\ y=-10 \end{cases}$$

The solution $x=-4$ and $y=-10$ is acceptable because $-10 \le -4 \le 0$.

c. $\begin{cases} y \le 0 \le x \\ x-y=6 \\ x-y=14 \end{cases}$ has no solution because it implies $6=14$.

You can try to consider the other three cases, and verify that you find the same solutions.