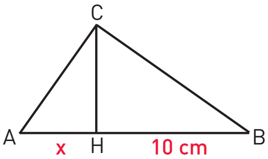
YOU & MATHS **Inequality on the sides** Let ABC be a right triangle with its right angle at C, with $AB = 10$ cm. Let CH be the altitude to the hypotenuse. Call $x = \overline{AH}$ and find the values of x such that:

$$\overline{CH} + \overline{AH} \geq 4.$$

Let us draw the triangle.



Using the right triangle altitude theorem, we get:

$$\overline{CH}^2 = \overline{AH} \cdot \overline{BH} = x \cdot (10 - x).$$

We have the constraints $0 \leq x \leq 10$, because x is a segment (therefore it has non-negative length) and is shorter than AB.
Therefore we can write

$$\overline{CH} = \sqrt{\overline{AH} \cdot \overline{BH}} = \sqrt{x \cdot (10 - x)},$$

because the radicand can not be negative.
We have to find the values of x such that $0 \leq x \leq 10$ and

$$\sqrt{x \cdot (10 - x)} + x \geq 4.$$

We do some calculations:

$$\sqrt{x \cdot (10 - x)} \geq 4 - x.$$

Now there are two cases:

a. if $x \geq 4$, then the inequality is true; since $x \leq 10$, in this case the solution to the inequality is $4 \leq x \leq 10$;

b. if $x < 4$, then we can raise both sides to the second power and obtain:

$$-x^2 + 10x \geq x^2 - 8x + 16 \rightarrow x^2 - 9x + 8 \leq 0.$$

Since $x < 4$, the solution to this inequality is:

$$1 \leq x < 4.$$

We put both cases together and we obtain the solution:

$$1 \leq x \leq 10.$$