

**YOU & MATHS** **Solutions in a range** Consider the following equation:

$$2mx - 3m = 5x - 9.$$

Find the values of  $m$  such that the absolute value of  $x$  is between 3 and 6, extreme values included.

Let us solve the equation for the variable  $x$ :

$$(2m - 5)x = 3m - 9.$$

If  $m = \frac{5}{2}$ , then no value of  $x$  is a solution, since  $0 \neq 3 \cdot \frac{5}{2} - 9 = -\frac{3}{2}$ .

If  $m \neq \frac{5}{2}$ , then:

$$x = \frac{3m - 9}{2m - 5} = 3 \cdot \frac{m - 3}{2m - 5}.$$

The absolute value of  $x$  is equal to:

$$|x| = 3 \cdot \left| \frac{m - 3}{2m - 5} \right|.$$

Then  $3 \leq |x| \leq 6$  is equivalent to:

$$3 \leq 3 \cdot \left| \frac{m - 3}{2m - 5} \right| \leq 6 \rightarrow 1 \leq \left| \frac{m - 3}{2m - 5} \right| \leq 2 \rightarrow |2m - 5| \leq |m - 3| \leq 2 \cdot |2m - 5|, \quad m \neq \frac{5}{2}.$$

Let us consider one inequality at a time, always remembering that  $m \neq \frac{5}{2}$ .

**a.**  $|2m - 5| \leq |m - 3|$ . Let us consider different cases.

- If  $m \geq 3$ , then the inequality is:

$$2m - 5 \leq m - 3;$$

we get  $m \leq 2$ , which is incompatible with  $m \geq 3$ .

There is no solution in this case.

- If  $\frac{5}{2} < m \leq 3$ , then the inequality is:

$$2m - 5 \leq 3 - m;$$

we get  $m \leq \frac{8}{3}$ .

The solution in this case is  $\frac{5}{2} < m \leq \frac{8}{3}$ .

- If  $m < \frac{5}{2}$ , then the inequality is:

$$-2m + 5 \leq 3 - m;$$

we get  $m \geq 2$ .

The solution in this case is  $2 \leq m < \frac{5}{2}$ .

When we consider the union of the solutions to the first inequality, we get:

$$2 \leq m < \frac{5}{2} \text{ or } \frac{5}{2} < m \leq \frac{8}{3},$$

which means:

$$2 \leq m \leq \frac{8}{3} \text{ and } m \neq \frac{5}{2}.$$

**b.**  $|m - 3| \leq |4m - 10|$

- If  $m \geq 3$ , then the inequality is:

$$m - 3 \leq 4m - 10;$$

we get  $m \geq \frac{7}{3}$ .

The solution in this case is  $m \geq 3$ .

- If  $\frac{5}{2} < m \leq 3$ , then the inequality is:

$$3 - m \leq 4m - 10;$$

we get  $m \geq \frac{13}{5}$ .

The solution in this case is  $m \geq \frac{13}{5}$ .

- If  $m < \frac{5}{2}$ , then the inequality is:

$$3 - m \leq 10 - 4m;$$

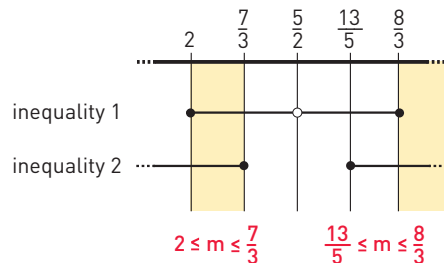
we get  $m \leq \frac{7}{3}$ .

The solution in this case is  $m \leq \frac{7}{3}$ .

When we consider the union of the solutions to the second inequality, we get:

$$m \geq \frac{13}{5} \text{ or } m \leq \frac{7}{3}.$$

When we consider the union of the solutions to the two inequalities we get what follows.



Therefore the solution to the pair of inequalities  $1 \leq \left| \frac{3m - 9}{2m - 5} \right| \leq 2$  is:

$$2 \leq m \leq \frac{7}{3} \text{ or } \frac{13}{5} \leq m \leq \frac{8}{3}.$$