YOU & MATHS   If a and b are roots of $2\sqrt{2x+4}-2=x$, then $ab=$

A   0   B   4   C   −12   D   −4   E   12

(USA *Tennessee Mathematics Teachers Association: 39th Annual Mathematics Contest*, 1995)

First of all, let us rearrange the terms of the equation as follows:

$$2\sqrt{2x+4}-2=x \ \rightarrow \ 2\sqrt{2x+4}=x+2.$$

We solve the irrational equation by setting up a linear system with a condition on the right-hand term:

$$\begin{cases} x+2 \geq 0 \\ 4(2x+4)=(x+2)^2 \end{cases} \rightarrow \begin{cases} x \geq -2 \\ 8x+16=x^2+4x+4 \end{cases} \rightarrow \begin{cases} x \geq -2 \\ x^2-4x-12=0 \end{cases}.$$

We compute the discriminant of the equation, find its solutions and check them with the condition in the system.

$$\frac{\Delta}{4}=\left(\frac{-4}{2}\right)^2-1(-12)=2^2+12=4+12=16$$

$$x_{1,2}=2\pm\sqrt{16}=2\pm4 \ \begin{smallmatrix} 6 \\ -2 \end{smallmatrix}$$

$$\begin{cases} x \geq -2 \\ x=-2 \vee x=6 \end{cases} \rightarrow \ x=-2 \vee x=6$$

The two roots of the irrational equation, a and b, are -2 and 6. Their product, ab, is equal to $(-2)\cdot6=-12$. Therefore our final answer is C.