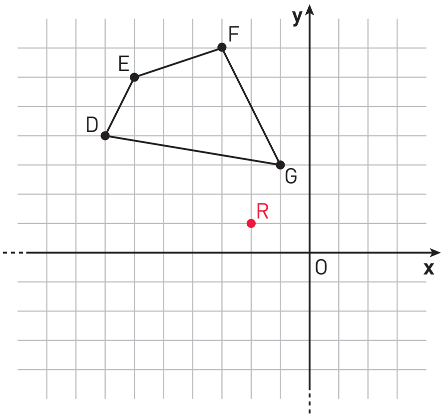
YOU & MATHS If quadrilateral $DEFG$ ($D(-7, 4)$, $E(-6, 6)$, $F(-3, 7)$, $G(-1, 3)$) is rotated 90° counterclockwise with center of rotation $(-2, 1)$, give the coordinates of each of the vertices of the image $D'E'F'G'$.

(USA *University of Houston: High School Mathematics Contest*, 2005)

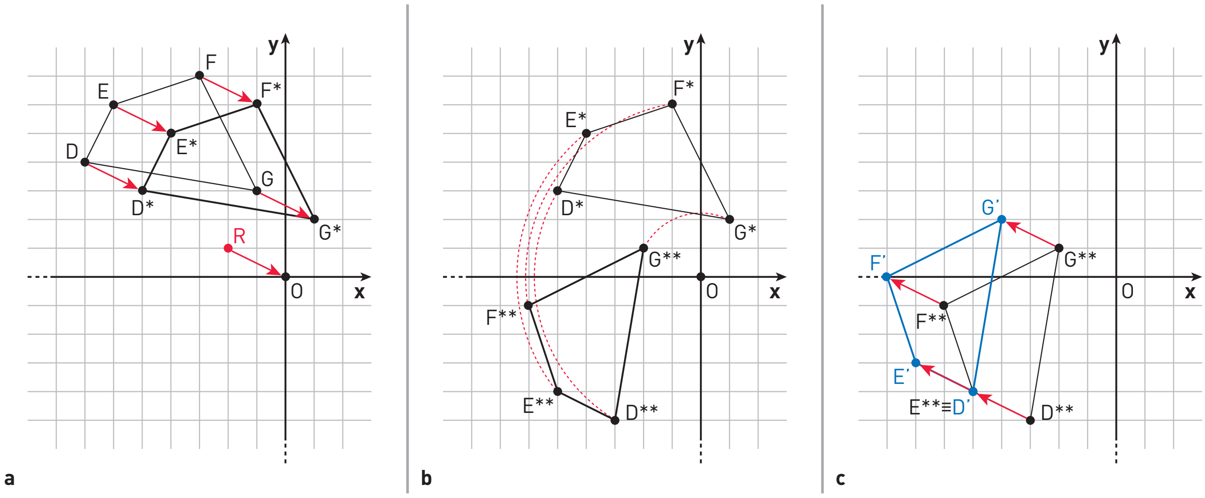
Let us call the centre of rotation R and let us draw the geometrical shape on the Cartesian plane.



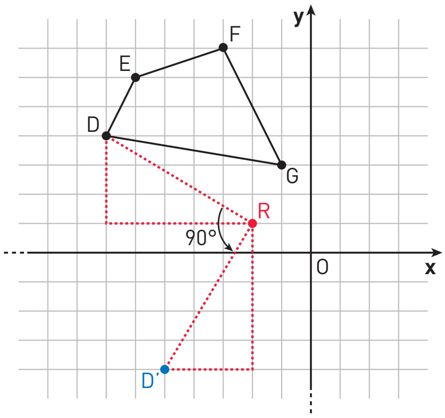
We recall that the equations of a 90° counterclockwise rotation of centre $O(0, 0)$ are

$$\begin{cases} x' = -y \\ y' = x \end{cases}.$$

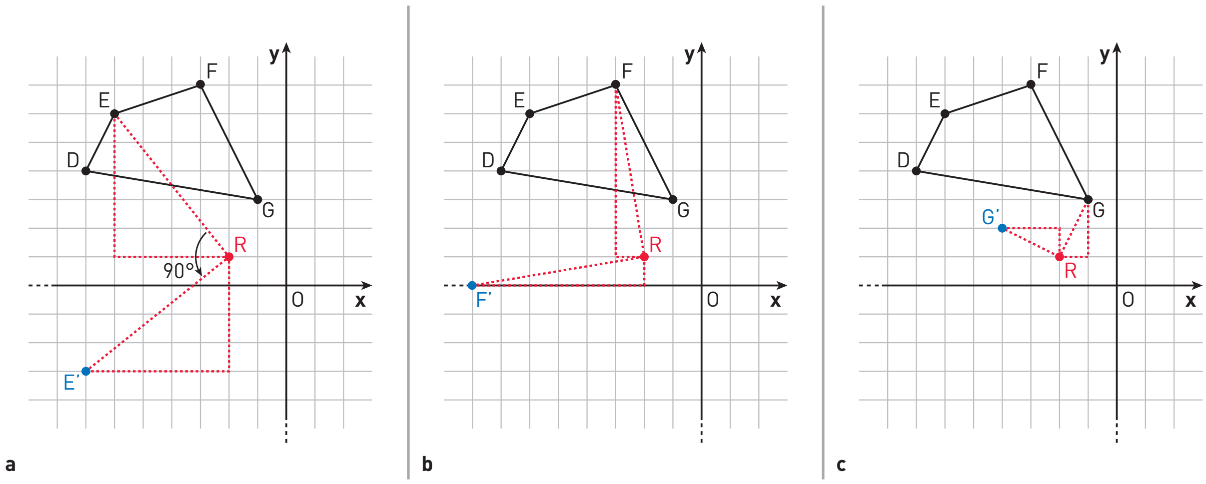
We can use these equations even if the centre of rotation is not the origin only if we first translate all the points along vector $\vec{v}(2, -1)$ (figure **a**). That is because, after this translation, point R will coincide with O. The vertices of the translated quadrilateral have coordinates $D^*(-5, 3)$, $E^*(-4, 5)$, $F^*(-1, 6)$, $G^*(1, 2)$.
We now apply the equations above for the rotation and obtain the coordinates of the rotated quadrilateral (figure **b**): $D^{**}(-3, -5)$, $E^{**}(-5, -4)$, $F^{**}(-6, -1)$, $G^{**}(-2, 1)$.
The last step to solve this problem is to translate the quadrilateral along vector $-\vec{v}$ in order to restore the initial conditions (figure **c**). By doing that, we obtain quadrilateral $D'E'F'G'$, which is the image of the 90° counterclockwise rotation of centre $(-2, 1)$ of $DEFG$. The vertices of $D'E'F'G'$ have the following coordinates: $D'(-5, -4)$, $E'(-7, -3)$, $F'(-8, 0)$, $G'(-4, 2)$.



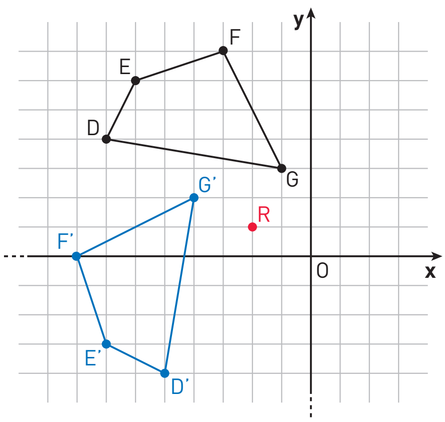
Alternatively, this problem can simply be solved graphically.
We first want to rotate point D onto point D' with a 90° counterclockwise rotation of centre R. To do that, we draw line segment DR and its perpendicular segment $D'R$, rotated in counterclockwise direction. $D'R$ has to be as long as DR and we can make sure that that is true by constructing a right triangle with hypotenuse DR and then rotating it so that the new hypotenuse is $D'R$, as in the figure on the next page.



We reiterate a similar procedure for points E, F and G, and find E', F' and G'.



The rotation of quadrilateral $DEFG$ around the centre of rotation R is the quadrilateral $D'E'F'G'$ shown below.



The coordinates of its vertices are: $D'(-5, -4)$, $E'(-7, -3)$, $F'(-8, 0)$, $G'(-4, 2)$.