

YOU & MATHS A regular polygon has each interior angle half as large as each exterior angle. How many sides does the polygon have?

- ☐ **A** 3 ☐ **D** 6
☐ **B** 4 ☐ **E** None of these answers.
☐ **C** 5

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The polygon of the problem is regular, which means that all its interior angles are congruent to each other and so are all its exterior angles.

If we call x the measure of each interior angle and y the measure of each exterior one, we have that

$$x = \frac{1}{2}y \quad \text{given.}$$

Moreover, an interior angle and an exterior one sum up to a straight angle, so

$$x + y = 180^\circ.$$

The two equations can be reunited and solved under a linear system, as follows:

$$\begin{cases} x = \frac{1}{2}y \\ x + y = 180^\circ \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}y \\ \frac{1}{2}y + y = 180^\circ \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}y \\ \frac{3}{2}y = 180^\circ \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}y \\ y = 120^\circ \end{cases} \rightarrow \begin{cases} x = 60^\circ \\ y = 120^\circ \end{cases}.$$

The only regular polygon that has interior angles that measure 60° is the equilateral triangle.