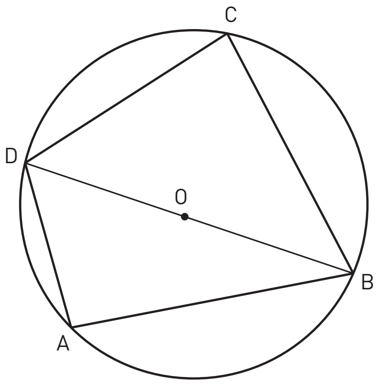
YOU & MATHS Consider a quadrilateral whose vertices, A, B, C, and D, are on a circle. Let x, y, and z be the truth values of the following three statements. What is the value of the ordered triple (x, y, z)?
x: For quadrilateral $ABCD$, $A\widehat{B}C + C\widehat{D}A = 180°$.
y: The perimeter of quadrilateral $ABCD$ is greater than twice the diameter of the circle.
z: The perpendicular bisector of any side will pass through the circle's center.

- **A** (F, F, T)
- **B** (F, T, T)
- **C** (T, T, T)
- **D** (F, F, F)
- **E** (T, F, T)

(USA *North Carolina State High School Mathematics Contest*, 2004)

Let us first sketch the quadrilateral $ABCD$ of the problem.



Its vertices lie on the circle, that is, the quadrilateral is inscribed in the circle and that implies that its opposite angles are supplementary. Since $A\widehat{B}C$ and $C\widehat{D}A$ are opposite, then it must be: $A\widehat{B}C + C\widehat{D}A = 180°$. Therefore, x is TRUE.

To enquire about the value of y, we consider the diameter BD of the circle corresponding to one of the diagonals of the quadrilateral. If we consider triangle BCD, by the triangle inequality, we have that

$$\overline{BC} + \overline{CD} > \overline{BD}.$$

Similarly, for triangle DAB, we have that

$$\overline{AB} + \overline{AD} > \overline{BD}.$$

By applying these two inequalities in the formula for the perimeter of the quadrilateral, we obtain:

$$2p(ABCD) = \overline{AB} + \overline{BC} + \overline{CD} + \overline{AD} > \overline{BD} + \overline{BD} = 2\overline{BD},$$

which tells us that y is TRUE.

Finally, we know that z is TRUE by the theorem that states that a polygon can be inscribed in a circle if and only if the perpendicular bisectors of its sides pass through the center of the circle.

As all three statements are true, our final answer is C.