If $2^4 \cdot 3^8 = n \cdot 6^4$, then $n =$

A 12 **B** 24 **C** 27 **D** 54 **E** 81

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To figure out the value of n, let's first rewrite the left-hand side of the equation using the definition of exponent. We have:

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2,$

$3^8 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3,$

so our equation becomes:

$2^4 \cdot 3^8 = n \cdot 6^4 \rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = n \cdot 6^4.$

We now notice that we can multiply some of the 2's and 3's on the left-hand side of the equation. By doing that, we get:

$(2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3) \cdot 3 \cdot 3 \cdot 3 \cdot 3 = n \cdot 6^4 \rightarrow 6 \cdot 6 \cdot 6 \cdot 6 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = n \cdot 6^4,$

which we can then write as:

$34 \cdot 6^4 = n \cdot 6^4.$

Because the term 6^4 appears on both sides of the equation, we have:

$n = 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81.$

The correct answer is therefore E.