YOU & MATHS **Can you simplify this?** Fully simplify $(1+x)(1+x^2)(1+x^4)(1+x^8)(1-x)$.

(USA *Texas A&M University Math Contest*, 2012)

Multiplication is commutative. So the product is also given by

$(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)$.

But $(a-b)(a+b) = a^2 - b^2$. So:

$(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8) =$

$(1-x^2)(1+x^2)(1+x^4)(1+x^8) =$

$(1-x^4)(1+x^4)(1+x^8) = (1-x^8)(1+x^8) = 1-x^{16}$.