YOU & MATHS **Another system to invent** Consider the system $\begin{cases} 2x + 3y + z = 5 \\ 4x + 9y + z = 25 \\ 8x + 27y + z = 125 \end{cases}$.

a. Is it dependent? Explain why or why not.

b. Invent another system of three equations in three unknowns that is dependent and that has the same right hand terms (5, 25, 125).

a. Let's calculate the determinant of the matrix of coefficients:

$$D = \begin{vmatrix} 2 & 3 & 1 \\ 4 & 9 & 1 \\ 8 & 27 & 1 \end{vmatrix} = 18 + 24 + 108 - (72 + 54 + 12) = 150 - 138 = 12 \neq 0.$$

Since the determinant is not zero, the system is consistent and independent.

b. A possible dependent system of three equations in three unknowns that has the same right hand terms (5; 25; 125) can be obtained multiplying all the terms of the first given equation, for example, by 5, and all the terms of the same equation by 25. We get a system with three proportional equations, and this system is, of course, dependent.

$$\begin{cases} 2x + 3y + z = 5 \\ 10x + 15y + 5z = 25 \\ 50x + 75y + 25z = 125 \end{cases}$$