YOU & MATHS P and Q are two points having coordinates $(-1, 3)$ and $(5, -1)$, respectively. Find:

a. slope of PQ;

b. K, the midpoint of PQ;

c. the equation of the line through K which is perpendicular to PQ. Test if this line contains the point $(4, 5)$.

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a. In order to find the slope of PQ, we use the formula for m and we substitute the coordinates of points P and Q:

$$m = \frac{y_Q - y_P}{x_Q - x_P} = \frac{-1-3}{5-(-1)} = \frac{-4}{6} = -\frac{2}{3}.$$

b. The midpoint K of PQ can be calculated using the midpoint formula:

$$K\left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2}\right) = K\left(\frac{-1+5}{2}, \frac{3+(-1)}{2}\right) = K\left(\frac{4}{2}, \frac{2}{2}\right) = K(2, 1).$$

c. The line perpendicular to PQ has slope

$$m' = -\frac{1}{m} = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}.$$

Its equation in slope-intercept form is then:

$$y = m'x + b \quad \rightarrow \quad y = \frac{3}{2}x + b.$$

We can easily find the value of the intercept b by substituting the coordinates of the point K into the equation, since the line passes through K. We get:

$$1 = \frac{3}{2}2 + b \quad \rightarrow \quad 1 = 3 + b \quad \rightarrow \quad b = -2.$$

The equation of the line through K and perpendicular to PQ is therefore:

$$y = \frac{3}{2}x - 2,$$

which can also be written as $y = 1.5x - 2$.

Finally, let us test if the line contains the point $(4, 5)$ by substituting its coordinates into the equation.

$$5 = 1.5 \cdot 4 - 2 \quad \rightarrow \quad 5 = 6 - 2 \quad \rightarrow \quad 5 = 4.$$

The equality does not hold, so we can conclude that the point $(4, 5)$ does not belong to the line.