

**YOU & MATHS**  $H$  is the point of coordinates  $(-2, 5)$  and  $K$  is the point  $(-2, -5)$ . Show that the  $x$ -axis bisects the line segment  $HK$ .  $T$  is the point such that the origin is the centre of  $HT$ . Find the coordinates of  $T$ . Verify that the  $y$ -axis bisects the line segment  $KT$ .

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- i. Let us show first that the  $x$ -axis bisects the line segment  $HK$ , which is equivalent to showing that the  $x$ -axis is perpendicular to  $HK$  and that it cuts  $HK$  in half.

The equation of the line through  $HK$  is:

$$x = -2,$$

as points  $H$  and  $K$  have the same  $x$ -coordinates. The equation of the  $x$ -axis is  $y = 0$ , so we see right away that the two lines are perpendicular.

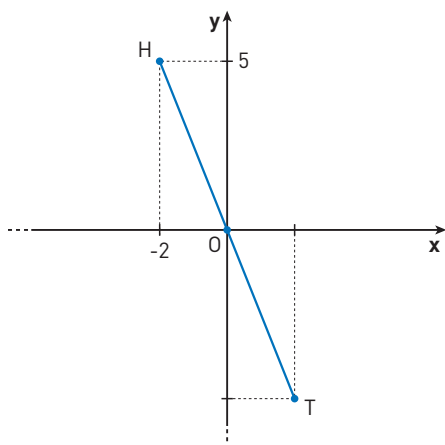
Moreover, the point of intersection between the  $x$ -axis and the line segment  $HK$  has coordinates  $(-2; 0)$ . If we call this point  $P$ , then:

$$\overline{HP} = |y_P - y_H| = |0 - 5| = 5;$$

$$\overline{PK} = |y_K - y_P| = |-5 - 0| = 5.$$

We can therefore conclude that the  $x$ -axis cuts the line segment  $HK$  in half.

- ii. Point  $T$  is symmetric to point  $H$  with respect to the origin, so its coordinates will be the opposite of the coordinates of  $H$ . Therefore we have  $T(2, -5)$ .



- iii. To verify that the  $y$ -axis bisects the line segment  $KT$ , we proceed as in i.

The equation of the line through  $KT$  is:

$$y = -5,$$

as points  $K$  and  $T$  have the same  $y$ -coordinate. The equation of the  $y$ -axis is  $x = 0$  and that makes it clear that the two lines are perpendicular.

Assuming that they intersect in point  $Q$ , which has coordinates  $(0, -5)$ , we have:

$$\overline{KQ} = |x_Q - x_K| = |0 - (-2)| = 2;$$

$$\overline{QT} = |x_T - x_Q| = |2 - 0| = 2.$$

We now know that the  $y$ -axis cuts the line segment  $KT$  in half and is perpendicular to it, so we can say that the  $y$ -axis bisects  $KT$ .