YOU & MATHS Find the value of y if $(6, y)$ lies on the same line as $(4, 6)$ and $(0, 4)$.

(USA *Southeast Missouri State University: Math Field Day*, 2005)

Let us calculate first the equation of the line through points $(4, 6)$ and $(0, 4)$. Using the formula

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1},$$

we substitute the coordinates of the points and get:

$$\frac{y - 6}{4 - 6} = \frac{x - 4}{0 - 4} \quad \rightarrow \quad \frac{y - 6}{-2} = \frac{x - 4}{-4} \quad \rightarrow \quad y - 6 = \frac{x - 4}{2} \quad \rightarrow \quad 2(y - 6) = x - 4 \quad \rightarrow$$

$$2y - 12 = x - 4 \quad \rightarrow \quad x - 2y + 8 = 0.$$

Now, to find the value of the y-coordinate of point $(6, y)$ we only need to substitute the value of its x-coordinate into the above equation:

$$6 - 2y + 8 = 0 \quad \rightarrow \quad -2y = -14 \quad \rightarrow \quad y = 7,$$

which is our final answer.