

YOU & MATHS Two students attempted to solve the quadratic equation $x^2 + bx + c = 0$. Although both students did the work correctly, one miscopied the middle term and obtained the solution set $\{2, 3\}$, while the other miscopied the constant term and obtained the solution set $\{2, 5\}$. What is the correct solution set? (USA Lehigh University: High School Math Contest, 2005)

The first student obtained the solution set $\{2, 3\}$, thus the quadratic equation $x^2 + bx + c = 0$ that he/she miscopied must have been of the form:

$$(x - 2)(x - 3) = 0,$$

which is equivalent to:

$$x^2 - 5x + 6 = 0.$$

We know that the middle term, -5 , was miscopied, so we can conclude that $c = 6$.

The second student got 2 and 5 as solutions. His/her equation then must have been:

$$(x - 2)(x - 5) = 0,$$

which is equivalent to:

$$x^2 - 7x - 10 = 0.$$

As the constant term was miscopied, we obtain that $b = -7$.

The original equation was then:

$$x^2 - 7x + 6 = 0,$$

whose discriminant is:

$$\Delta = b^2 - 4ac = 7^2 - 4 \cdot 6 = 49 - 24 = 25.$$

The two correct solutions are:

$$x_{1,2} = \frac{7 \pm \sqrt{25}}{2} = \frac{7 \pm 5}{2} = \begin{matrix} 6 \\ 1 \end{matrix}$$

or equivalently, $\{1, 6\}$.