**YOU & MATHS** **The vertex at the origin** Find the values of $a$ and $b$ such that the parabola

$$y = ax^2 - (a-1)x + b$$

has its vertex at the origin.

The equation $y = ax^2 - (a-1)x + b$ defines a parabola if and only if $a \neq 0$.
Let $x_V$ be the $x$-coordinate of the vertex of the parabola. We know that:

$$x_V = \frac{a-1}{2a}.$$

It has to be $x_V = 0$, so

$$0 = \frac{a-1}{2a} \quad \rightarrow \quad a = 1.$$

The equation of the parabola then becomes:

$$y = x^2 - (1-1)x + b = -x^2 + b.$$

To find $y_V$ we substitute $x_V = 0$ in the equation of the parabola:

$$y_V = -x_V^2 + b = 0 + b = b.$$

It has to be $y_V = 0$, so $b = 0$.

The solution is then $a = 1$, $b = 0$.