

YOU & MATHS Sums and cubes Two numbers add up to -5 . The cubes of the two numbers add up to -35 . Find the two numbers.

Let x, y be the two numbers. We know that:

$$\begin{cases} x + y = -5 \\ x^3 + y^3 = -35 \end{cases}$$

Let us do some calculations:

$$\begin{cases} x = -y - 5 \\ (-y - 5)^3 + y^3 = -35 \end{cases} \rightarrow \begin{cases} x = -y - 5 \\ -y^3 - 15y^2 - 75y - 125 + y^3 = -35 \end{cases} \rightarrow \begin{cases} x = -y - 5 \\ y^2 + 5y + 6 = 0 \end{cases}$$

The solutions of the equation $y^2 + 5y + 6 = 0$ are:

$$y = -3 \text{ and } y = -2.$$

If $y = -3$, then $x = -y - 5 = -2$.

If $y = -2$, then $x = -y - 5 = -3$.

Therefore there is only one pair of numbers that solves the problem, and the two numbers are -2 and -3 .