

YOU & MATHS Solve the inequality $\frac{(x-2)(x+5)}{(x-3)(x+3)} \geq 0$.

Write the solution in interval notation.

(USA Southeast Missouri State University: Math Field Day, 2005)

In order to solve the rational inequality, we study the sign of both the numerator and the denominator of the fraction by setting them greater than or equal to zero.

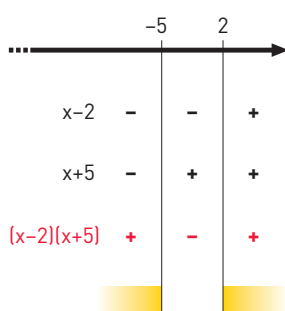
- $(x-2)(x+5) \geq 0$

The numerator is made up of two factors, whose sign we are going to study separately.

$$x-2 \geq 0 \rightarrow x \geq 2$$

$$x+5 \geq 0 \rightarrow x \geq -5$$

By looking at the diagram below, we conclude that the numerator is greater than or equal to zero when $x \leq -5 \vee x \geq 2$.



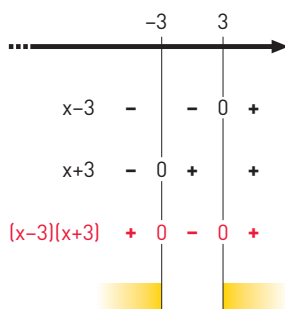
- $(x-3)(x+3) > 0$

The denominator is made up of two factors as well, so we are going to consider this inequality as a factorial one.

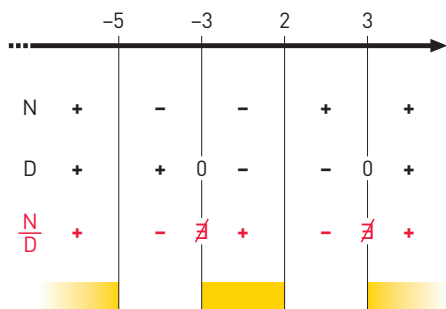
$$x-3 > 0 \rightarrow x > 3$$

$$x+3 > 0 \rightarrow x > -3$$

By looking at the diagram below, we can say that the denominator is greater than zero when $x < -3 \vee x > 3$.



Finally, we put the results we have found for the numerator and the denominator in another diagram to compare their signs.



While recalling to pay attention to those values that make the denominator equal to zero, we get the final solution:

$$x \leq -5 \vee -3 < x \leq 2 \vee x > 3,$$

which can be written in interval notation as

$$]-\infty; -5] \cup]-3; 2] \cup]3; +\infty[$$