

**YOU & MATHS** The set of real numbers satisfying  $\frac{1}{x+1} > \frac{1}{x-2}$  is:

- A**  $\{x \mid x > 2\}$ .                       **D**  $\{x \mid x < -1\}$ .  
 **B**  $\{x \mid -1 < x < 2\}$ .                 **E**  $\{x \mid x > -1\}$ .  
 **C**  $\{x \mid x < 2\}$ .

(USA North Carolina State High School Mathematics Contest, 2003)

The first step to solve a rational inequality is to move all its terms to the left-hand side so that only the number zero is left on the right-hand side. Let us do that:

$$\frac{1}{x+1} > \frac{1}{x-2} \rightarrow \frac{1}{x+1} - \frac{1}{x-2} > 0.$$

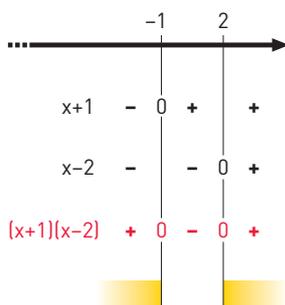
By subtracting the two fractions, we get:

$$\frac{x-2-(x+1)}{(x+1)(x-2)} > 0 \rightarrow \frac{x-2-x-1}{(x+1)(x-2)} \rightarrow \frac{-3}{(x+1)(x-2)} > 0.$$

Let us study the signs of both the numerator and the denominator.

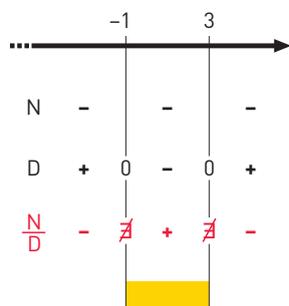
- The numerator is always negative, as  $-3 < 0$ .
- The denominator is made up of two factors, so we are going to study it as a factorial inequality:

$$\begin{aligned} (x+1)(x-2) &> 0 \\ x+1 > 0 &\rightarrow x > -1 \\ x-2 > 0 &\rightarrow x > 2 \end{aligned}$$



The denominator is positive for  $x < -1 \vee x > 2$ .

Let us put together our results for the numerator and denominator.



The final result is then  $-1 < x < 2$ , that is, the solution of our inequality is the set of real numbers  $\{x \mid -1 < x < 2\}$ . We therefore pick answer B.