

YOU & MATHS For all real numbers p, q, x, y which satisfy $x > p$ and $y > q$, which of the following inequalities are satisfied?

1. $x^2y^2 > p^2q^2$ 2. $x + y > p + q$ 3. $x^2 + y^2 > p^2 + q^2$

A 1, 2, 3

B 1 only

C 2, 3

D 2 only

E None

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The fact that $x > p$ and $y > q$ implies that:

$$x + y > p + y > p + q, \text{ for all real values of } x, y, p \text{ and } q.$$

Unfortunately, since we do not know whether these four unknowns are positive or negative, we cannot prove the validity of the other two inequalities. For instance:

- if $x > p > 0$ and $y > q > 0$ (e.g. $x = y = 2$ and $p = q = 1$), then $x^2y^2 > p^2q^2$ and $x^2 + y^2 > p^2 + q^2$ (inequalities 1 and 2) hold;
but
- if $x < 0, p < 0, y < 0$ and $q < 0$ (e.g. $x = y = -1$ and $p = q = -2$), then $x^2y^2 < p^2q^2$ and $x^2 + y^2 < p^2 + q^2$ are verified!

Therefore our final answer is D.