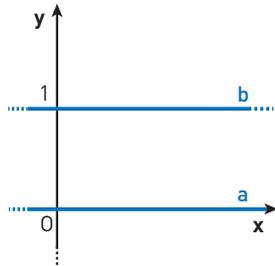


YOU & MATHS Distances from two lines Draw two parallel lines a, b . Let $k > 0$ be a number. Find all the points P of the plane such that the distance from P to line b is equal to k times the distance from P to line a .

Let us draw the two parallel lines a and b in a Cartesian coordinate system; let us choose the origin and the axes so that the equation of line a is $y = 0$ and the equation of line b is $y = 1$.



For any point $P(x, y)$ on the plane, the distances from P to the two lines a and b are:

$$d(P, a) = |y| \text{ and } d(P, b) = |y - 1|.$$

We want to find the points whose coordinates satisfy the equality:

$$|y - 1| = k \cdot |y|.$$

We notice that the coordinate x of point P is not involved in the equation; this means that the solution of the problem is a locus formed by one or several lines that are parallel to the x -axis.

The equation is an absolute value equation, so we split it into two cases using inequalities:

1. $0 \leq y \leq 1$. Then the equation is:

$$1 - y = k \cdot y; y = \frac{1}{1 + k}.$$

In particular, if $k = 1$ we get

$$y = \frac{1}{2}.$$

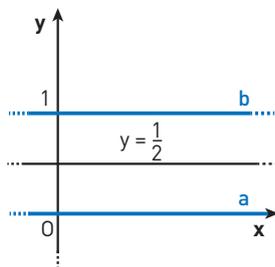
2. $y > 1$ or $y < 0$. Then the equation is:

$$y - 1 = k \cdot y.$$

If $k = 1$, there is no solution. If $k \neq 1$, then the equation is:

$$y = \frac{1}{1 - k}.$$

In conclusion, if $k = 1$, the solution is represented by the line $y = \frac{1}{2}$.



If $k \neq 1$, the solution is given by the points of the two lines $y = \frac{1}{1+k}$ and $y = \frac{1}{1-k}$.

For example, if $k = \frac{1}{2}$.

