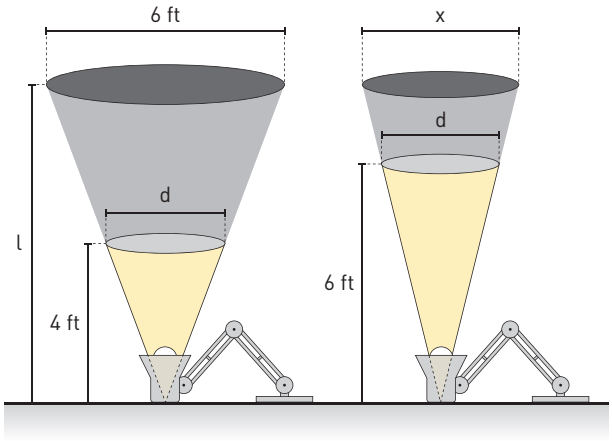
YOU & MATHS In a normally-shaped room, there is a light on the floor. If I hold a disc 4 feet above the light, then it casts a circular shadow on the ceiling of diameter 6 feet. If I then raise the disc 2 feet, what will be the new diameter of the shadow on the ceiling?

(USA *Bay Area Math Meet, BAMM, Bowl Sampler,* 1995)

Let us call d the diameter of the disc and l the distance between the light and the ceiling.



Looking at the situation from the side, we notice that the cone of light identifies two similar triangles: one that has the diameter of the disc as a base, the other one that has the diameter of the shadow on the ceiling as a base. The angles adjacent to the bases of the triangles are all congruent as they are correspondent angles of two parallel lines (the two bases) cut by a transversal line (the edges of the cone of light). Therefore, the two triangles are similar by the AA similarity criterion.
That implies the following proportion:

$$\frac{4}{l} = \frac{d}{6} \quad \rightarrow \quad d = \frac{24}{l}.$$

After we raise the disc 2 feet, the distance from the lamp to the disc increases to 6 feet. Following the same reasoning as before, the two new triangles are similar by the AA similarity criterion and they allow us to state that:

$$\frac{6}{l} = \frac{d}{x},$$

where we indicated by x the diameter of the new shadow.
The diameter of the disc d and the distance from the lamp to the ceiling l have not changed, so we can substitute the expression we have found earlier for d in the latter equation. We get:

$$\frac{6}{l} = \frac{\frac{24}{l}}{x} \quad \rightarrow \quad x = \frac{24}{l} \cdot \frac{l}{6} = 4.$$

Therefore the diameter of the new shadow on the ceiling measures 4 feet.