YOU & MATHS A circle of greatest area is cut out of a 4 cm square of material. A square of greatest area is then cut out of the circle. How much material is wasted?

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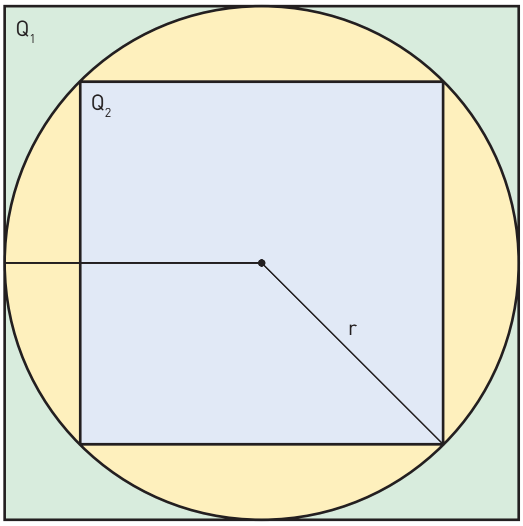

First of all, let us notice that the area of the initial square of material is:

$$\text{area}(Q_1) = 4^2 = 16$$

in square centimetres.
The greatest circle than can be cut out of it is the circle inscribed in the square, which has radius half as long as the side of the square, that is, $r = 2$ cm.
The square of the greatest area that can be cut out of the circle is the square inscribed in the circle, which has diagonal twice as long as the radius of the circle itself, that is, $d = 2r = 4$ cm. As all squares are rhombuses, we use the formula for the area of a rhombus to calculate the area of the inner square:

$$\text{area}(Q_2) = \frac{d^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

in square centimetres.
The area of the material that has been cut out is equal to the area of the initial square of material minus the area of the smaller inscribed square:

$$\text{area}(Q_1) - \text{area}(Q_2) = 16 - 8 = 8.$$

in square centimetres. Therefore 8 cm^2 of material are wasted.