YOU & MATHS Suppose there is a seven-sided die with the property that, when it is rolled, there is an equal probability of getting 1, 2, 3, 4, 5, 6, or 7. If this die is rolled twice, then what is the probability that the sum of the two rolls is even?

(USA *Bay Area Math Meet*, *BAMM, Bowl Sampler,* 1995)

The event E, whose probability we want to calculate, is defined as:

$E = \{\text{sum of two rolls of die is even}\}$.

As the sum of two numbers is even only when the numbers are either both even or both odd, we can define the two events as follows:

$E_1 = \{\text{both rolls are even numbers}\}$ and $E_2 = \{\text{both rolls are odd numbers}\}$.

The two events are incompatible, so the probability of E is given by the sum of the probabilities of E_1 and E_2. Since every roll of a die is an independent event, we apply the multiplication rule and we get:

$p(E_1) = p(1^{\text{st}} \text{ roll is an even number}) \cdot p(2^{\text{nd}} \text{ roll is an even number}) =$

$p(1^{\text{st}} \text{ roll is 2, 4 or 6}) \cdot p(2^{\text{nd}} \text{ roll is 2, 4 or 6}) = \frac{3}{7} \cdot \frac{3}{7} = \frac{9}{49}$

$p(E_2) = p(1^{\text{st}} \text{ roll is an odd number}) \cdot p(2^{\text{nd}} \text{ roll is an odd number}) =$

$p(1^{\text{st}} \text{ roll is 1, 3, 5 or 7}) \cdot p(2^{\text{nd}} \text{ roll is 1, 3, 5 or 7}) = \frac{4}{7} \cdot \frac{4}{7} = \frac{16}{49}$.

We can finally calculate the probability that the sum of the two rolls is even:

$p(E) = p(E_1) + p(E_2) = \frac{9}{49} + \frac{16}{49} = \frac{25}{49}$.