YOU & MATHS A bag contains a number of marbles of which 80 are red, 24 are white and the rest are blue. If the probability of randomly selecting a blue marble from this bag is $\frac{1}{5}$, how many blue marbles are there in the bag?

A 25 B 26 C 27 D 28 E 29

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First of all, let us recall that, according to its classical definition, the probability of an event is given by the number of successful outcomes divided by the total number of possible outcomes.

The probability of randomly selecting a blue marble from the bag is $\frac{1}{5}$ (given in the text of the problem), but it can also be calculated as

$$p(E) = \frac{\text{number of marbles}}{\text{total number of marbles}} = \frac{\text{number of blue marbles}}{\text{number of red marbles} + \text{number of white marbles} + \text{number of blue marbles}}.$$

If we assume that x is equal to the number of blue marbles and we substitute the number of red and white marbles in the fraction, we get the equation

$$\frac{x}{80 + 24 + x} = \frac{1}{5}.$$

Let us solve it for x:

$$\frac{x}{104 + x} = \frac{1}{5} \rightarrow \frac{5x}{5(104 + x)} = \frac{104 + x}{5(104 + x)}.$$

We set the condition on the denominator $x \neq -104$ (which is impossible in our case as x represents the number of marbles) and then we get rid of it by multiplying both sides of the equation by $5(104 + x)$.
We obtain:

$$5x = 104 + x \rightarrow 4x = 104 \rightarrow x = 26.$$

The number of blue marbles in the bag is 26. The correct answer is B.