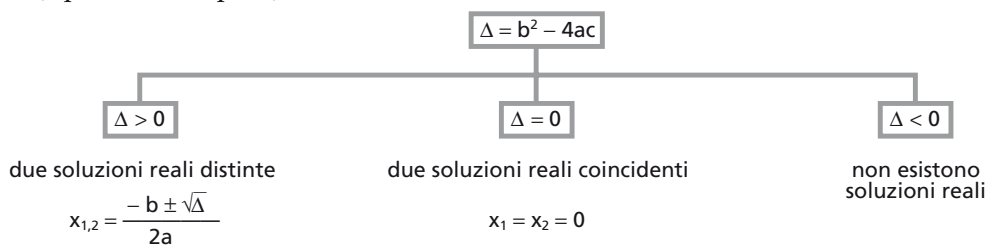


RICHIAMI DI ALGEBRA

LE EQUAZIONI DI SECONDO GRADO

Un'equazione di secondo grado è riconducibile alla forma normale: $ax^2 + bx + c = 0, a \neq 0$

- $b = 0, c \neq 0$ (equazione pura) $\rightarrow ax^2 + c = 0 \rightarrow x^2 = -\frac{c}{a}$
 - se $-\frac{c}{a} < 0$: impossibile
 - se $-\frac{c}{a} > 0 \rightarrow x_{1,2} = \pm \sqrt{-\frac{c}{a}}$
- $c = 0, b \neq 0$ (equazione spuria) $\rightarrow ax^2 + bx = 0 \rightarrow x(ax + b) = 0 \rightarrow x_1 = 0, x_2 = -\frac{b}{a}$
- $b = c = 0$ (equazione monomia) $\rightarrow ax^2 = 0 \rightarrow x_1 = x_2 = 0$
- $b \neq 0, c \neq 0$ (equazione completa). Il discriminante è $\Delta = b^2 - 4ac$.



Formula ridotta: b pari $\rightarrow x_{1,2} = \frac{-\frac{b}{2} \pm \sqrt{\frac{\Delta}{4}}}{a}$.

LE DISEQUAZIONI DI SECONDO GRADO

Per risolvere le disequazioni $ax^2 + bx + c > 0$ e $ax^2 + bx + c < 0$ (con $a > 0$), si considera l'equazione associata $ax^2 + bx + c = 0$.

Se $\Delta > 0$, la disequazione:

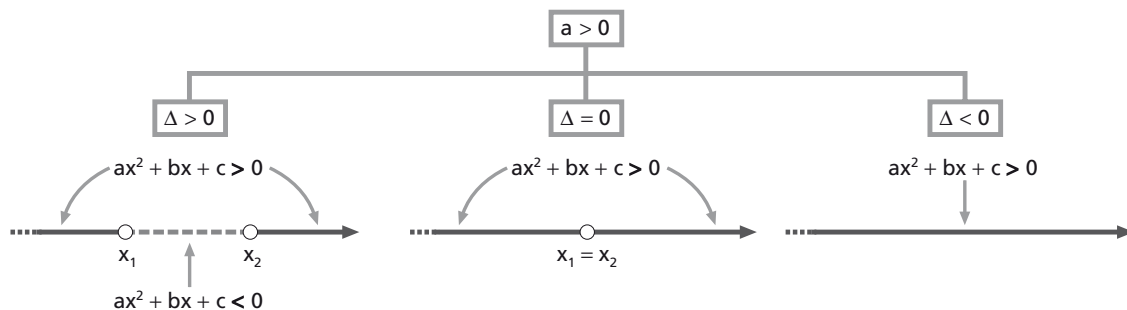
- $ax^2 + bx + c > 0$ è verificata dai valori esterni all'intervallo individuato dalle radici dell'equazione associata;
- $ax^2 + bx + c < 0$ è verificata dai valori interni.

Se $\Delta = 0$, la disequazione:

- $ax^2 + bx + c > 0$ è sempre verificata tranne che per il valore della radice doppia dell'equazione associata;
- $ax^2 + bx + c < 0$ non è mai verificata.

Se $\Delta < 0$, la disequazione:

- $ax^2 + bx + c > 0$ è sempre verificata;
- $ax^2 + bx + c < 0$ non è mai verificata.



LE EQUAZIONI E LE DISEQUAZIONI CON IL VALORE ASSOLUTO

$$|A(x)| = k \begin{cases} \text{se } k < 0: \text{ non ha soluzione} \\ \text{se } k \geq 0: A(x) = \pm k \end{cases}$$

$$|A(x)| < k \begin{cases} \text{se } k > 0: -k < A(x) < k \rightarrow \begin{cases} A(x) > -k \\ A(x) < k \end{cases} \\ \text{se } k \leq 0: \text{ non ha soluzione} \end{cases}$$

$$|A(x)| > k \begin{cases} \text{se } k > 0: A(x) < -k \vee A(x) > k \\ \text{se } k = 0: A(x) \neq 0 \\ \text{se } k < 0: \text{ sempre verificata} \end{cases}$$

LE EQUAZIONI E LE DISEQUAZIONI IRRAZIONALI

$$\sqrt[n]{A(x)} = B(x) \begin{cases} \text{se } n \text{ è dispari: } A(x) = [B(x)]^n \\ \text{se } n \text{ è pari: } \begin{cases} A(x) \geq 0 \\ B(x) \geq 0 \\ A(x) = [B(x)]^n \end{cases} \end{cases}$$

$$\sqrt[n]{A(x)} < B(x) \begin{cases} \text{se } n \text{ è dispari: } A(x) < [B(x)]^n \\ \text{se } n \text{ è pari: } \begin{cases} A(x) \geq 0 \\ B(x) > 0 \\ A(x) < [B(x)]^n \end{cases} \end{cases}$$

$$\sqrt[n]{A(x)} > B(x) \begin{cases} \text{se } n \text{ è dispari: } A(x) > [B(x)]^n \\ \text{se } n \text{ è pari: } \begin{cases} B(x) < 0 \\ A(x) \geq 0 \end{cases} \vee \begin{cases} B(x) \geq 0 \\ A(x) > [B(x)]^n \end{cases} \end{cases}$$

LE PROPRIETÀ DELLE POTENZE

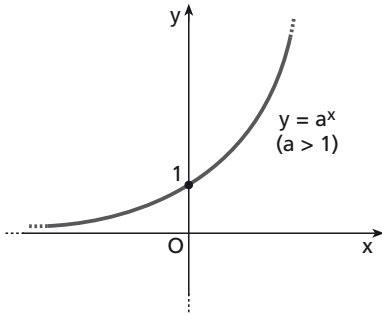
- $a^m \cdot a^n = a^{m+n}$
- $a^m : a^n = a^{m-n}$ con $a \neq 0$
- $(a^m)^n = a^{m \cdot n}$
- $a^m \cdot b^m = (a \cdot b)^m$
- $a^m : b^m = (a : b)^m$ con $b \neq 0$
- $a^{-n} = \frac{1}{a^n}$ con $a \neq 0$

I PRODOTTI NOTEVOLI E LA SCOMPOSIZIONE IN FATTORI

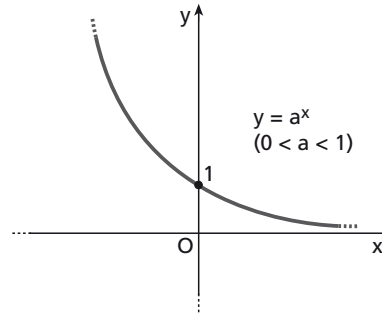
- $(A + B)(A - B) = A^2 - B^2$
- $(A \pm B)^2 = A^2 \pm 2AB + B^2$
- $(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$
- $(A \pm B)^3 = A^3 \pm 3A^2B + 3AB^2 \pm B^3$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$

LA FUNZIONE ESPONENZIALE E LA FUNZIONE LOGARITMO

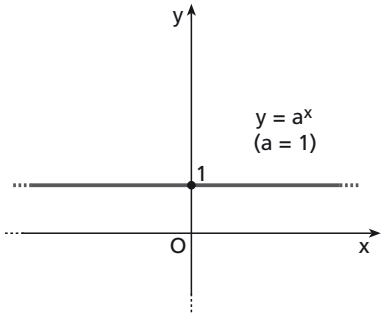
La funzione esponenziale



- a. • C.E.: \mathbb{R} ;
 • codominio: \mathbb{R}^+ ;
 • funzione crescente in \mathbb{R} ;
 • corrispondenza biunivoca;
 • $a^x \rightarrow 0$ per $x \rightarrow -\infty$;
 • $a^x \rightarrow +\infty$ per $x \rightarrow +\infty$.

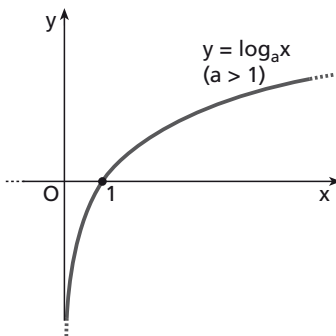


- b. • C.E.: \mathbb{R} ;
 • codominio: \mathbb{R}^+ ;
 • funzione decrescente in \mathbb{R} ;
 • corrispondenza biunivoca;
 • $a^x \rightarrow 0$ per $x \rightarrow +\infty$;
 • $a^x \rightarrow +\infty$ per $x \rightarrow -\infty$.

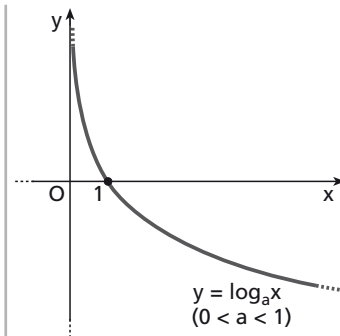


- c. • C.E.: \mathbb{R} ;
 • codominio: $\{1\}$;
 • funzione costante;
 • funzione non iniettiva.

La funzione logaritmo



- a. • C.E.: \mathbb{R}^+ ;
 • codominio: \mathbb{R} ;
 • funzione crescente in \mathbb{R}^+ ;
 • corrispondenza biunivoca;
 • $\log_a x \rightarrow -\infty$ per $x \rightarrow 0$;
 • $\log_a x \rightarrow +\infty$ per $x \rightarrow +\infty$.



- b. • C.E.: \mathbb{R}^+ ;
 • codominio: \mathbb{R} ;
 • funzione decrescente in \mathbb{R}^+ ;
 • corrispondenza biunivoca;
 • $\log_a x \rightarrow +\infty$ per $x \rightarrow 0$;
 • $\log_a x \rightarrow -\infty$ per $x \rightarrow +\infty$.

Logaritmo di un prodotto

$$\log_a (b \cdot c) = \log_a b + \log_a c, \quad (b > 0, c > 0)$$

Logaritmo di un quoziente

$$\log_a \frac{b}{c} = \log_a b - \log_a c, \quad (b > 0, c > 0)$$

Logaritmo di una potenza

$$\log_a b^n = n \cdot \log_a b, \quad (b > 0)$$

Cambiamento di base nei logaritmi

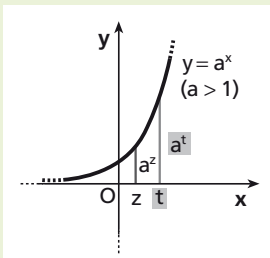
$$\log_a b = \frac{\log_c b}{\log_c a} \quad a > 0, b > 0, c > 0$$

$$a \neq 1, c \neq 1$$

Disequazioni esponenziali

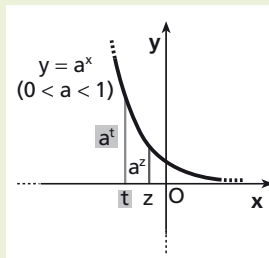
$$a > 1$$

$$a^t > a^z \Leftrightarrow t > z$$



$$0 < a < 1$$

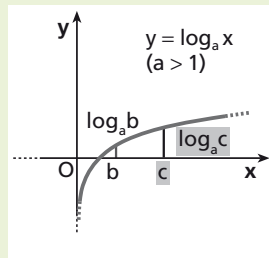
$$a^t > a^z \Leftrightarrow t < z$$



Disequazioni logaritmiche

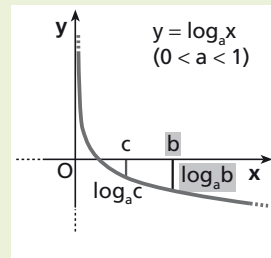
$$a > 1$$

$$\log_a b < \log_a c \Leftrightarrow b < c$$



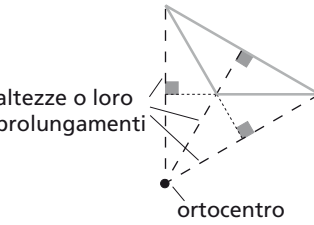
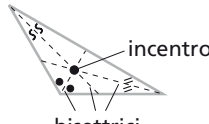
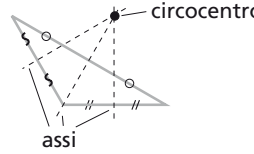
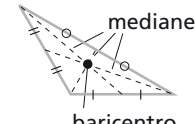
$$0 < a < 1$$

$$\log_a b < \log_a c \Leftrightarrow b > c$$



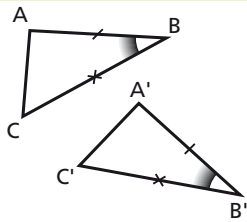
RICHIAMI DI GEOMETRIA

I punti notevoli di un triangolo

 <p>altezze o loro prolungamenti</p> <p>ortocentro</p>	 <p>bisettrici</p> <p>incentro</p> <p>L'incentro è il centro della circonferenza inscritta.</p>	 <p>assi</p> <p>circocentro</p> <p>Il circocentro è il centro della circonferenza circoscritta.</p>	 <p>mediane</p> <p>baricentro</p> <p>Il baricentro divide ogni mediana in due parti di cui quella contenente il vertice è doppia dell'altra</p>
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I criteri di congruenza dei triangoli

1° criterio



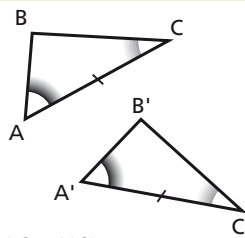
$$AB \cong A'B'$$

$$BC \cong B'C'$$

$$\hat{A} \cong \hat{A}' \Rightarrow ABC \cong A'B'C'$$

$$\hat{B} \cong \hat{B}'$$

2° criterio

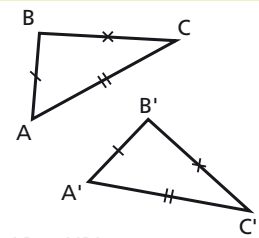


$$AC \cong A'C'$$

$$\hat{A} \cong \hat{A}' \Rightarrow ABC \cong A'B'C'$$

$$\hat{C} \cong \hat{C}'$$

3° criterio



$$AB \cong A'B'$$

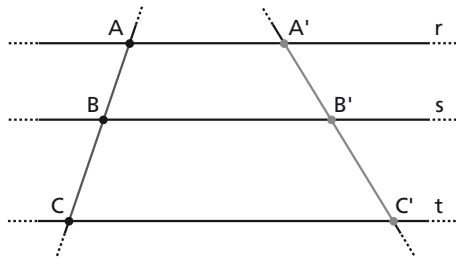
$$BC \cong B'C'$$

$$\hat{B} \cong \hat{B}' \Rightarrow ABC \cong A'B'C'$$

$$AC \cong A'C'$$

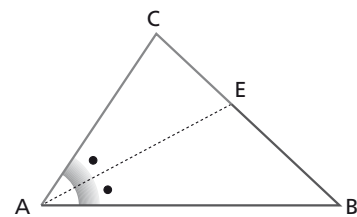
Il teorema di Talete

Teorema di Talete



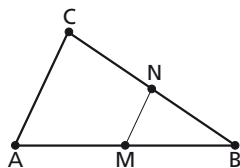
$$r \parallel s \parallel t \Rightarrow AB : BC = A'B' : B'C'$$

Teorema della bisettrice di un angolo interno di un triangolo



$$BE : CE = AB : AC$$

Conseguenza del teorema di Talete



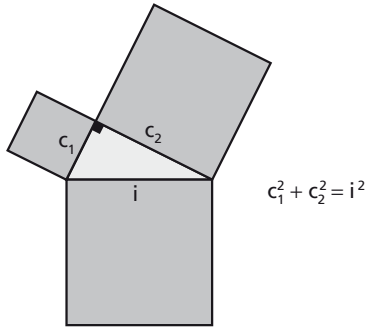
$$AM \cong MB$$

$$CN \cong NB \Rightarrow MN \parallel AC$$

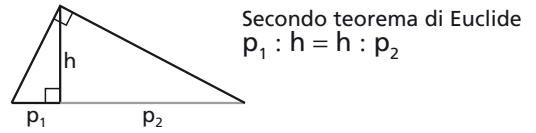
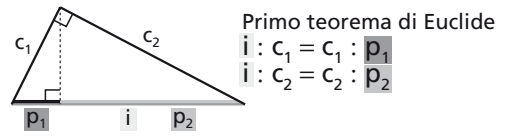
$$MN \cong \frac{1}{2} AC$$

L'equivalenza e la similitudine

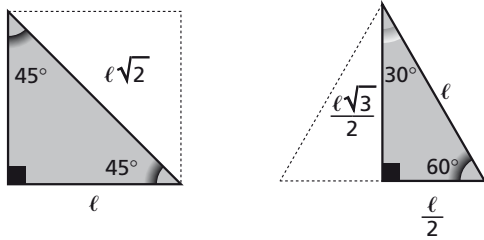
Il teorema di Pitagora



I teoremi di Euclide



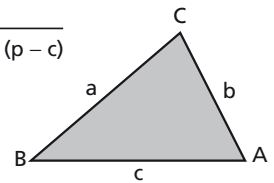
Relazioni fra i lati di triangoli notevoli



Formula di Erone

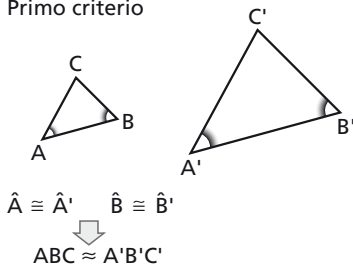
$$s = \sqrt{p \cdot (p - a) \cdot (p - b) \cdot (p - c)}$$

con $p = \frac{a + b + c}{2}$

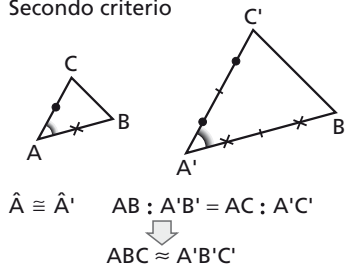


Criteri di similitudine dei triangoli

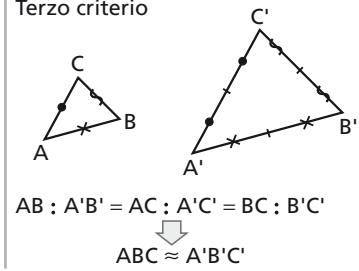
Primo criterio



Secondo criterio

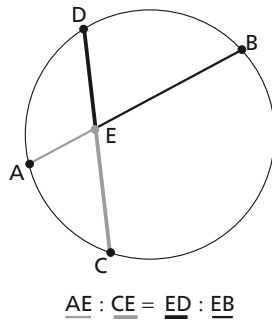


Terzo criterio

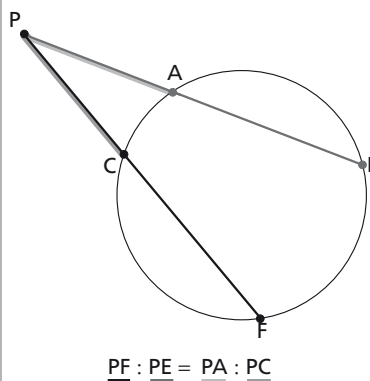


La similitudine nella circonferenza

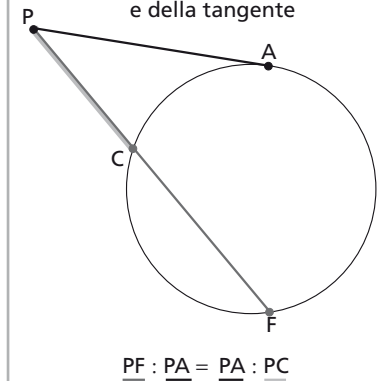
Teorema delle corde secanti



Teorema delle secanti

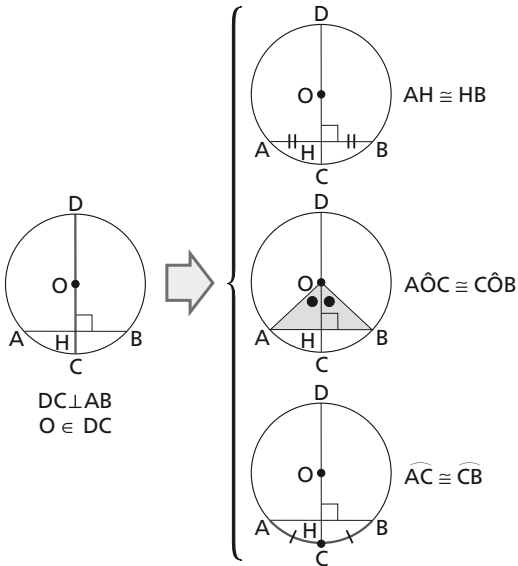
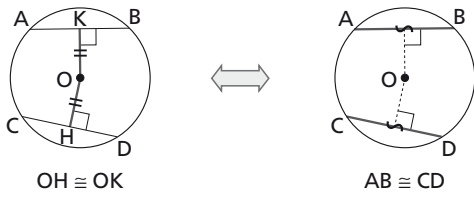


Teorema della secante e della tangente

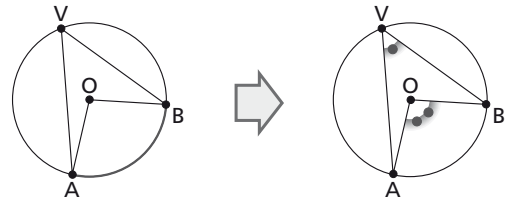


La circonferenza

I teoremi sulle corde

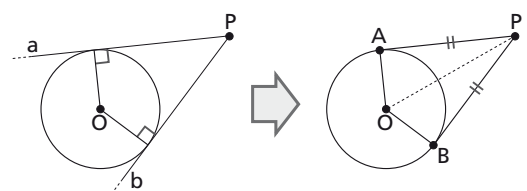


Angoli alla circonferenza e angoli al centro



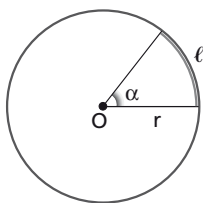
Ogni angolo alla circonferenza è la metà dell'angolo al centro corrispondente.

Tangente a una circonferenza da un punto esterno



Se da un punto esterno a una circonferenza si conducono le rette tangenti, i segmenti di tangente risultano congruenti

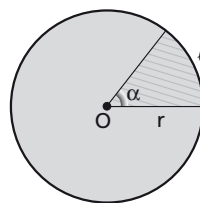
La lunghezza della circonferenza e l'area del cerchio



$$c = 2\pi r$$

$$\ell = \frac{\alpha}{180} \pi r$$

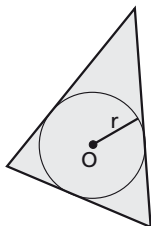
a. Misure della circonferenza (c) e dell'arco di angolo al centro α (ℓ).



$$C = \pi r^2$$

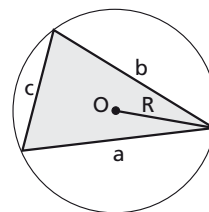
$$S = \frac{\alpha}{360} \pi r^2 = \frac{1}{2} \ell r$$

b. Misure dell'area del cerchio (C) e dell'area del settore circolare di angolo al centro α (S).



$$r = \frac{A}{p}$$

a. Raggio del cerchio inscritto nel triangolo.

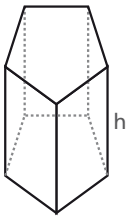


$$R = \frac{abc}{4A}$$

b. Raggio del cerchio circoscritto al triangolo.

Formule di geometria solida

PRISMA RETTO

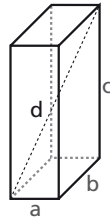


$$A_l = 2p \cdot h$$

$$A_t = A_l + 2A_b$$

$$V = A_b \cdot h$$

PARALLELEPIPEDO RETTANGOLO



$$A_b = ab$$

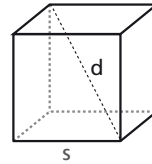
$$A_l = 2(ac + bc)$$

$$A_t = 2(ac + ab + bc)$$

$$V = a \cdot b \cdot c$$

$$d = \sqrt{a^2 + b^2 + c^2}$$

CUBO



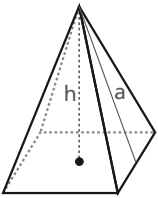
$$A_b = s^2$$

$$A_l = 6s^2$$

$$V = s^3$$

$$d = s\sqrt{3}$$

PIRAMIDE RETTA

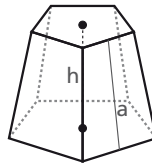


$$A_b = p \cdot a$$

$$A_l = A_l + A_b$$

$$V = \frac{1}{3} A_b \cdot h$$

TRONCO DI PIRAMIDE RETTA

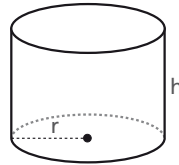


$$A_b = (p + p') \cdot a$$

$$A_l = A_l + A_b + A'_b$$

$$V = \frac{1}{3} h (A_b + A'_b + \sqrt{A_b \cdot A'_b})$$

CILINDRO



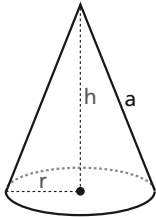
$$A_b = \pi r^2$$

$$A_l = 2\pi r \cdot h$$

$$A_t = 2\pi r (h + r)$$

$$V = \pi r^2 \cdot h$$

CONO



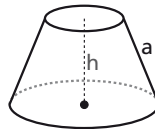
$$A_b = \pi r^2$$

$$A_l = \pi r a$$

$$A_t = \pi r (a + r)$$

$$V = \frac{1}{3} \pi r^2 \cdot h$$

TRONCO DI CONO



$$A_b = \pi r^2$$

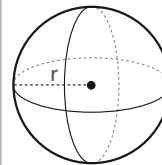
$$A'_b = \pi r'^2$$

$$A_l = \pi a (r + r')$$

$$A_t = A_l + A_b + A'_b$$

$$V = \frac{1}{3} \pi h (r^2 + r'^2 + r \cdot r')$$

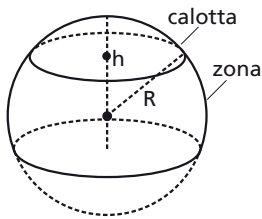
SFERA



$$A = 4\pi r^2$$

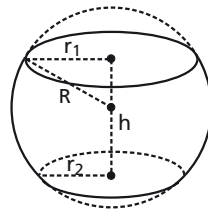
$$V = \frac{4}{3} \pi r^3$$

CALOTTA E ZONA SFERICA



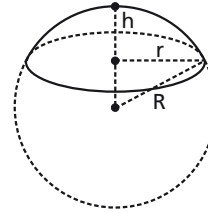
$$S = 2\pi R h$$

SEGMENTO SFERICO A DUE BASI



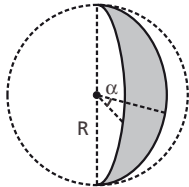
$$V = \frac{4}{3} \pi \left(\frac{h}{2}\right)^3 + \pi r_1^2 \frac{h}{2} + \pi r_2^2 \frac{h}{2}$$

SEGMENTO SFERICO A UNA BASE



$$V = \frac{4}{3} \pi \left(\frac{h}{2}\right)^3 + \pi r^2 \frac{h}{2} = \frac{1}{3} \pi h^2 (3R - h)$$

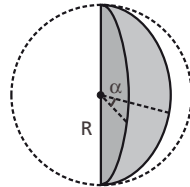
FUSO SFERICO



$$S_f = 2R^2 \alpha^{rad} = \frac{\alpha^\circ}{90^\circ} \pi R^2$$

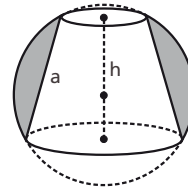
α^{rad} : ampiezza del diedro in radianti
 α° : ampiezza del diedro in gradi

SPICCHIO SFERICO



$$V_s = \frac{2}{3} \alpha^{rad} R^3 = \frac{\alpha^\circ}{270^\circ} \pi R^3$$

ANELLO SFERICO



$$V_a = \frac{1}{6} \pi a^2 h$$

GEOMETRIA ANALITICA

La **distanza fra due punti** $A(x_A; y_A)$ e $B(x_B; y_B)$ è data da: $\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$.

Il **punto medio** del segmento AB è $M(x_M; y_M)$ con: $x_M = \frac{x_A + x_B}{2}$, $y_M = \frac{y_A + y_B}{2}$.

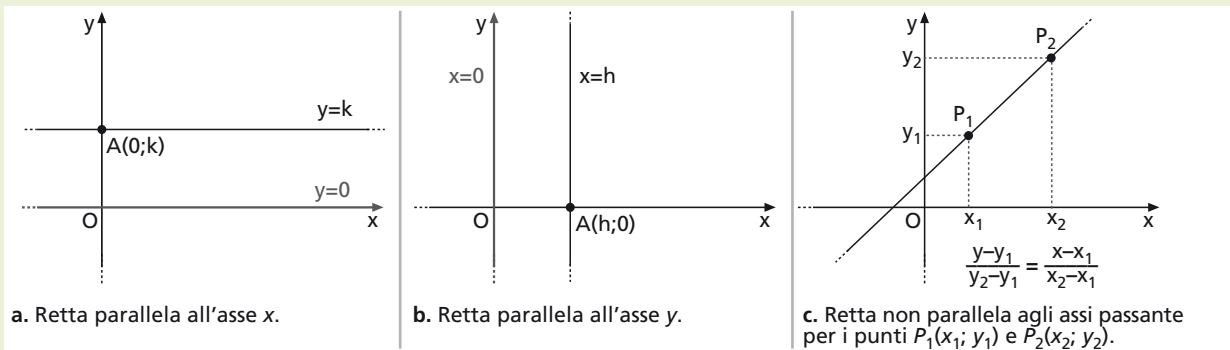
Il **baricentro di un triangolo** di vertici $A(x_A; y_A)$, $B(x_B; y_B)$, $C(x_C; y_C)$ è $G(x_G; y_G)$ con:

$$x_G = \frac{x_A + x_B + x_C}{3}, \quad y_G = \frac{y_A + y_B + y_C}{3}.$$

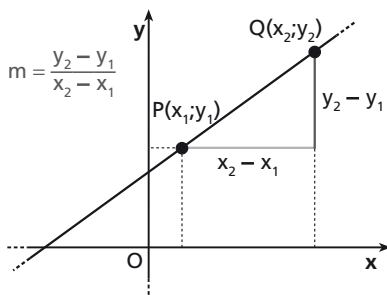
La **distanza di un punto** $P(x_0; y_0)$ **da una retta** r di equazione $ax + by + c = 0$ è uguale a: $d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.

Il piano cartesiano e la retta

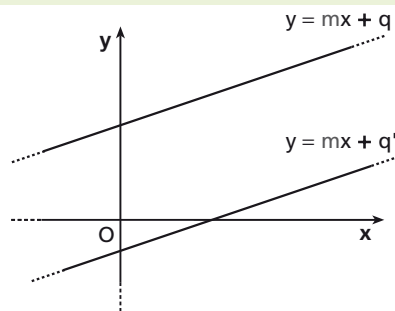
L'equazione di una retta



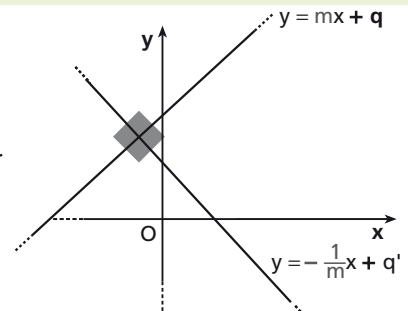
Coefficiente angolare



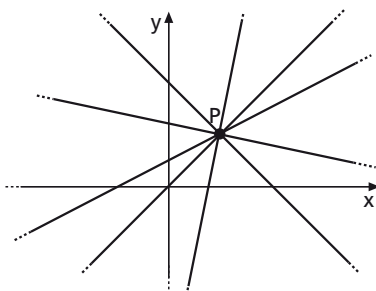
Rette parallele



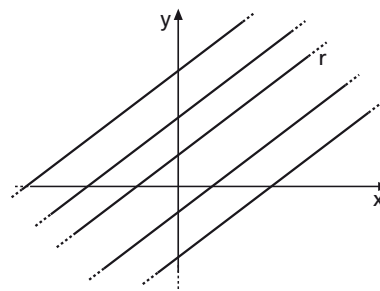
Rette perpendicolari



I fasci di rette



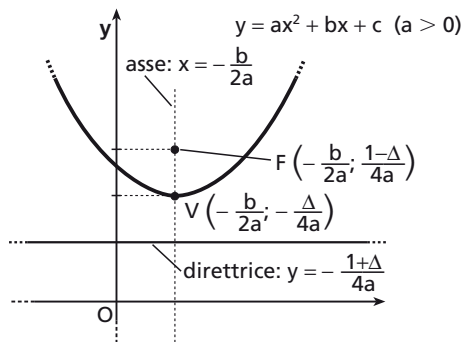
a. **Fascio proprio** di rette per un punto P : insieme di tutte le rette del piano passanti per P . P è detto **centro del fascio**.



b. **Fascio improprio** di rette parallele a una retta r .

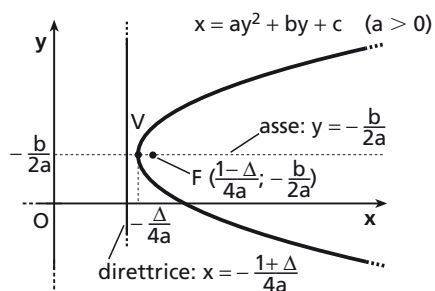
Le coniche

La parabola con asse parallelo all'asse y



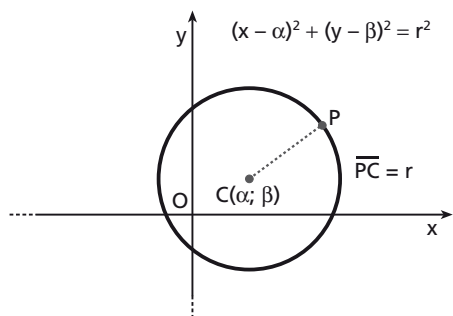
se $a < 0$ la concavità è rivolta verso il basso

La parabola con asse parallelo all'asse x

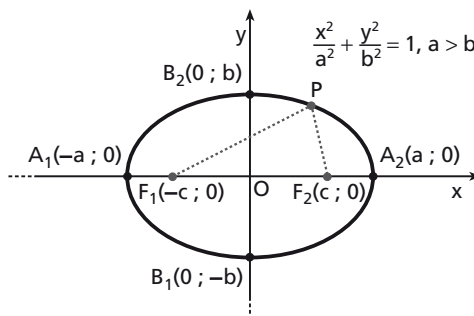


se $a < 0$ la concavità è rivolta nel verso opposto

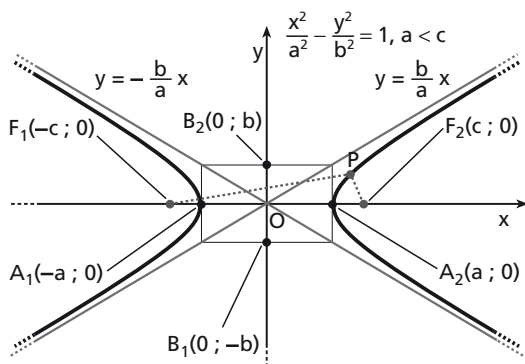
La circonferenza



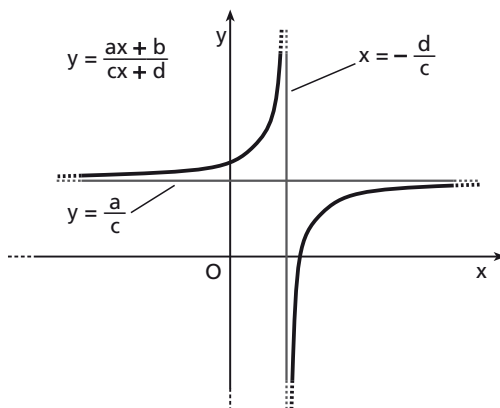
L'ellisse



L'iperbole

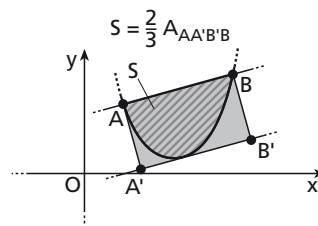


La funzione omografica



IL SEGMENTO PARABOLICO

Tracciamo la retta parallela ad AB e tangente alla parabola, e consideriamo su di essa le proiezioni A' e B' di A e B . L'area del segmento parabolico è uguale a $\frac{2}{3}$ dell'area del rettangolo $AA'B'B$.



LA SIMMETRIA ASSIALE

Fissata nel piano una retta r , la **simmetria assiale rispetto alla retta r** è quella isometria che a ogni punto del piano P fa corrispondere il punto P' del semipiano opposto rispetto a r , in modo che r sia l'asse del segmento PP' , ossia:

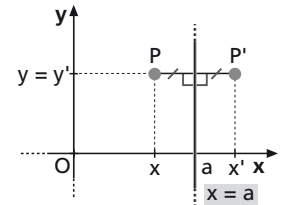
- r passa per il punto medio di PP' ;
- PP' è perpendicolare alla retta r .

La retta r è detta **asse di simmetria**.

Nel piano cartesiano prendiamo in esame le seguenti simmetrie assiali, fornendo le relative equazioni.

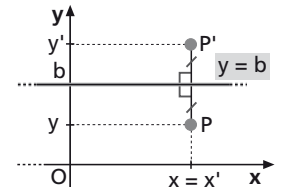
a. Simmetria con asse $x = a$ (asse parallelo all'asse y)

$$\begin{cases} x' = 2a - x \\ y' = y \end{cases}$$



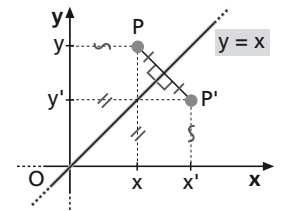
b. Simmetria con asse $y = b$ (asse parallelo all'asse x)

$$\begin{cases} x' = x \\ y' = 2b - y \end{cases}$$



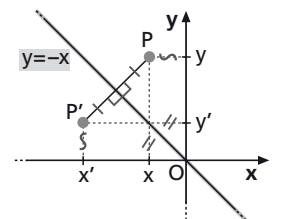
c. Simmetria con asse $y = x$ (bisettrice del primo e terzo quadrante)

$$\begin{cases} x' = y \\ y' = x \end{cases}$$



d. Simmetria con asse $y = -x$ (bisettrice del secondo e quarto quadrante)

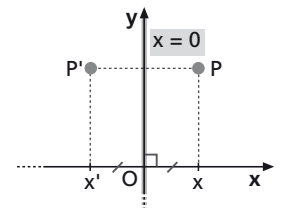
$$\begin{cases} x' = -y \\ y' = -x \end{cases}$$



e. Simmetria con asse $x = 0$ (asse y)

$$\begin{cases} x' = -x \\ y' = y \end{cases}$$

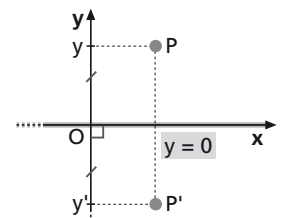
Due punti simmetrici rispetto all'asse y hanno ascisse opposte e la stessa ordinata.



f. Simmetria con asse $y = 0$ (asse x)

$$\begin{cases} x' = x \\ y' = -y \end{cases}$$

Due punti simmetrici rispetto all'asse x hanno la stessa ascissa e ordinate opposte.



GONIOMETRIA E TRIGONOMETRIA

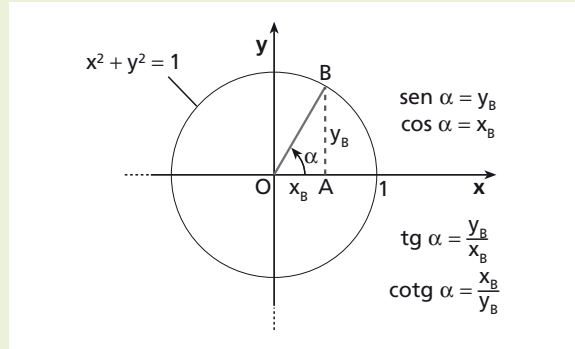
Le funzioni goniometriche

La prima relazione fondamentale

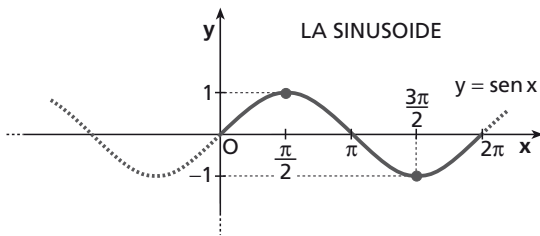
$$\sin^2 \alpha + \cos^2 \alpha = 1$$

La seconda relazione fondamentale

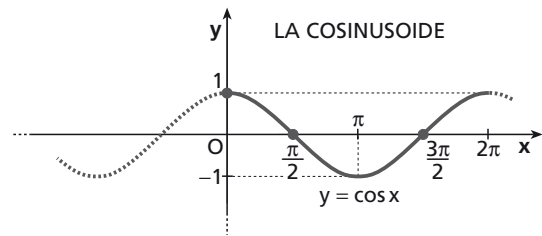
$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$



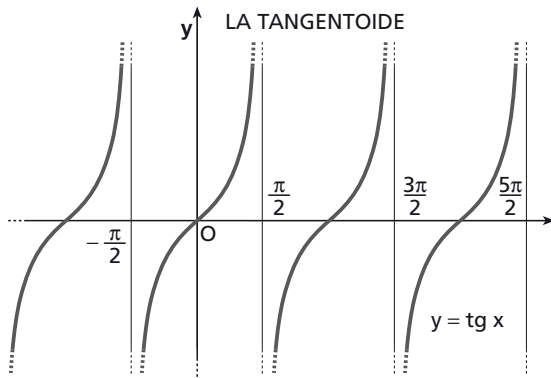
I grafici delle funzioni goniometriche



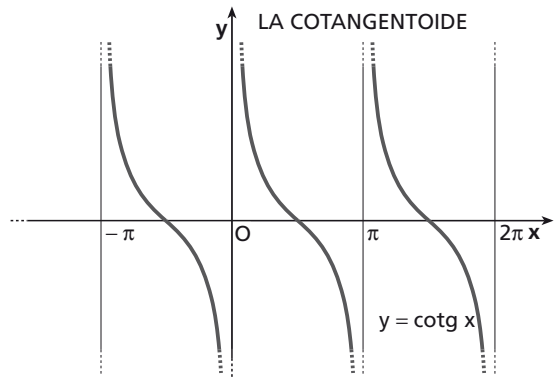
Periodicità: $\forall \alpha \in \mathbb{R}, \forall k \in \mathbb{Z} \quad \sin(\alpha + 2k\pi) = \sin \alpha$



Periodicità: $\forall \alpha \in \mathbb{R}, \forall k \in \mathbb{Z} \quad \cos(\alpha + 2k\pi) = \cos \alpha$



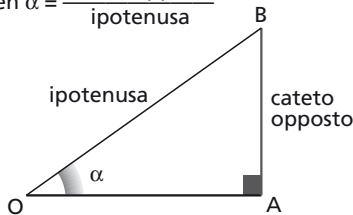
Periodicità: $\forall \alpha \in \mathbb{R}, \forall k \in \mathbb{Z} \quad \operatorname{tg}(\alpha + k\pi) = \operatorname{tg} \alpha$



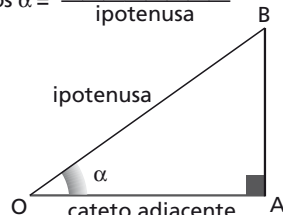
Periodicità: $\forall \alpha \in \mathbb{R}, \forall k \in \mathbb{Z} \quad \operatorname{cotg}(\alpha + k\pi) = \operatorname{cotg} \alpha$

Senò, coseno e tangente su un triangolo rettangolo

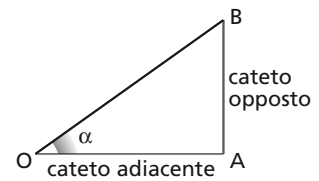
$$\sin \alpha = \frac{\text{cateto opposto}}{\text{ipotenusa}}$$



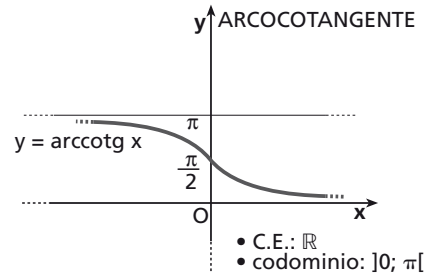
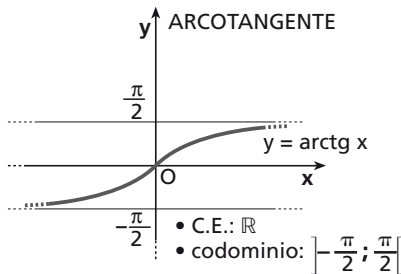
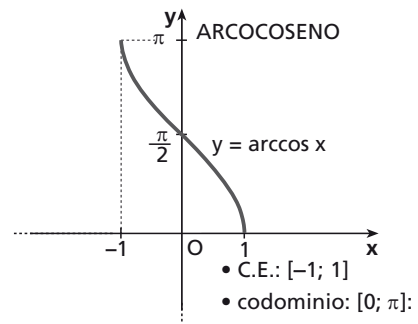
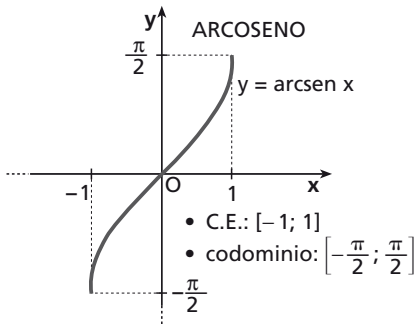
$$\cos \alpha = \frac{\text{cateto adiacente}}{\text{ipotenusa}}$$



$$\operatorname{tg} \alpha = \frac{\text{cateto opposto}}{\text{cateto adiacente}}$$



Le funzioni goniometriche inverse

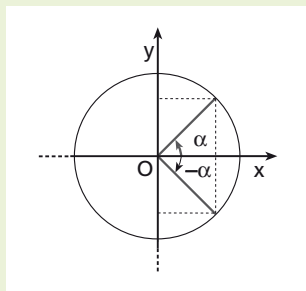


Seno, coseno, tangente e cotangente di angoli notevoli

radianti	gradi	seno	coseno	tangente	cotangente
0	0	0	1	0	non esiste
$\frac{\pi}{12}$	15°	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$2 - \sqrt{3}$	$2 + \sqrt{3}$
$\frac{\pi}{10}$	18°	$\frac{\sqrt{5} - 1}{4}$	$\frac{\sqrt{10 + 2\sqrt{5}}}{4}$	$\frac{\sqrt{25 - 10\sqrt{5}}}{5}$	$\sqrt{5 + 2\sqrt{5}}$
$\frac{\pi}{8}$	22°30'	$\frac{\sqrt{2} - \sqrt{2}}{2}$	$\frac{\sqrt{2 + \sqrt{2}}}{2}$	$\sqrt{2} - 1$	$\sqrt{2} + 1$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{5}$	36°	$\frac{\sqrt{10 - 2\sqrt{5}}}{4}$	$\frac{\sqrt{5} + 1}{4}$	$\sqrt{5 - 2\sqrt{5}}$	$\frac{\sqrt{25 + 10\sqrt{5}}}{5}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{3}{10} \pi$	54°	$\frac{\sqrt{5} + 1}{4}$	$\frac{\sqrt{10 - 2\sqrt{5}}}{4}$	$\frac{\sqrt{25 + 10\sqrt{5}}}{5}$	$\sqrt{5 - 2\sqrt{5}}$
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{2}{5} \pi$	72°	$\frac{\sqrt{10 + 2\sqrt{5}}}{4}$	$\frac{\sqrt{5} - 1}{4}$	$\sqrt{5 + 2\sqrt{5}}$	$\frac{\sqrt{25 - 10\sqrt{5}}}{5}$
$\frac{5}{12} \pi$	75°	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$2 + \sqrt{3}$	$2 - \sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	non esiste	0

Funzioni goniometriche di angoli associati

α e $-\alpha$



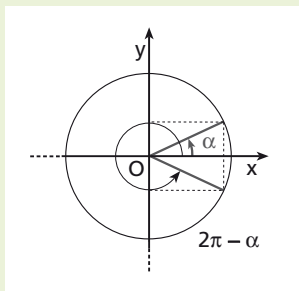
$$\text{sen}(-\alpha) = -\text{sen} \alpha$$

$$\text{cos}(-\alpha) = \text{cos} \alpha$$

$$\text{tg}(-\alpha) = -\text{tg} \alpha$$

$$\text{cotg}(-\alpha) = -\text{cotg} \alpha$$

α e $2\pi - \alpha$



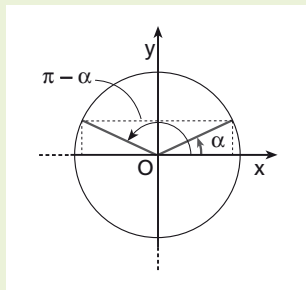
$$\text{sen}(2\pi - \alpha) = -\text{sen} \alpha$$

$$\text{cos}(2\pi - \alpha) = \text{cos} \alpha$$

$$\text{tg}(2\pi - \alpha) = -\text{tg} \alpha$$

$$\text{cotg}(2\pi - \alpha) = -\text{cotg} \alpha$$

α e $\pi - \alpha$



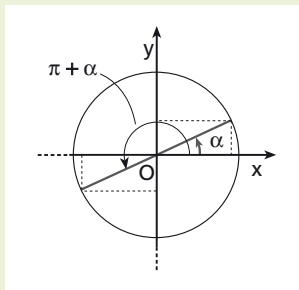
$$\text{sen}(\pi - \alpha) = \text{sen} \alpha$$

$$\text{cos}(\pi - \alpha) = -\text{cos} \alpha$$

$$\text{tg}(\pi - \alpha) = -\text{tg} \alpha$$

$$\text{cotg}(\pi - \alpha) = -\text{cotg} \alpha$$

α e $\pi + \alpha$



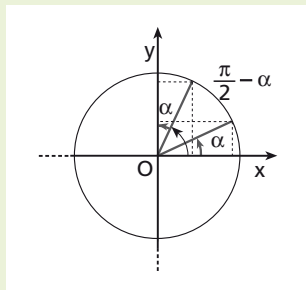
$$\text{sen}(\pi + \alpha) = -\text{sen} \alpha$$

$$\text{cos}(\pi + \alpha) = -\text{cos} \alpha$$

$$\text{tg}(\pi + \alpha) = \text{tg} \alpha$$

$$\text{cotg}(\pi + \alpha) = \text{cotg} \alpha$$

α e $\frac{\pi}{2} - \alpha$



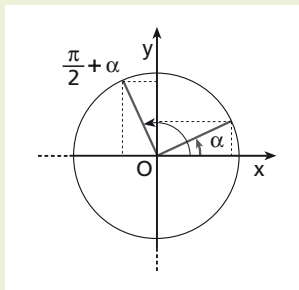
$$\text{sen}\left(\frac{\pi}{2} - \alpha\right) = \text{cos} \alpha$$

$$\text{cos}\left(\frac{\pi}{2} - \alpha\right) = \text{sen} \alpha$$

$$\text{tg}\left(\frac{\pi}{2} - \alpha\right) = \text{cotg} \alpha$$

$$\text{cotg}\left(\frac{\pi}{2} - \alpha\right) = \text{tg} \alpha$$

α e $\frac{\pi}{2} + \alpha$



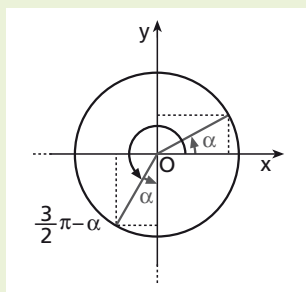
$$\text{sen}\left(\frac{\pi}{2} + \alpha\right) = \text{cos} \alpha$$

$$\text{cos}\left(\frac{\pi}{2} + \alpha\right) = -\text{sen} \alpha$$

$$\text{tg}\left(\frac{\pi}{2} + \alpha\right) = -\text{cotg} \alpha$$

$$\text{cotg}\left(\frac{\pi}{2} + \alpha\right) = -\text{tg} \alpha$$

α e $\frac{3}{2}\pi - \alpha$



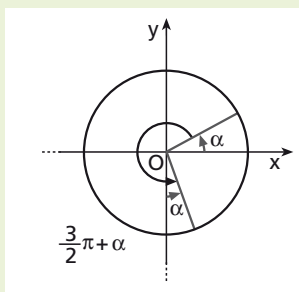
$$\text{sen}\left(\frac{3}{2}\pi - \alpha\right) = -\text{cos} \alpha$$

$$\text{cos}\left(\frac{3}{2}\pi - \alpha\right) = -\text{sen} \alpha$$

$$\text{tg}\left(\frac{3}{2}\pi - \alpha\right) = \text{cotg} \alpha$$

$$\text{cotg}\left(\frac{3}{2}\pi - \alpha\right) = \text{tg} \alpha$$

α e $\frac{3}{2}\pi + \alpha$



$$\text{sen}\left(\frac{3}{2}\pi + \alpha\right) = -\text{cos} \alpha$$

$$\text{cos}\left(\frac{3}{2}\pi + \alpha\right) = \text{sen} \alpha$$

$$\text{tg}\left(\frac{3}{2}\pi + \alpha\right) = -\text{cotg} \alpha$$

$$\text{cotg}\left(\frac{3}{2}\pi + \alpha\right) = -\text{tg} \alpha$$

Le formule goniometriche

Le formule di addizione

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$\cos \alpha, \beta, \alpha + \beta \neq \frac{\pi}{2} + k\pi$$

Le formule di sottrazione

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$\cos \alpha, \beta, \alpha - \beta \neq \frac{\pi}{2} + k\pi$$

Le formule di duplicazione

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

Le formule di bisezione

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

Le formule parametriche

$$\sin \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}, \cos \alpha \neq \pi + k2\pi$$

Le formule di prostaferesi

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$$

$$\sin p - \sin q = 2 \cos \frac{p+q}{2} \cdot \sin \frac{p-q}{2}$$

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \cdot \sin \frac{p-q}{2}$$

Le formule di Werner

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

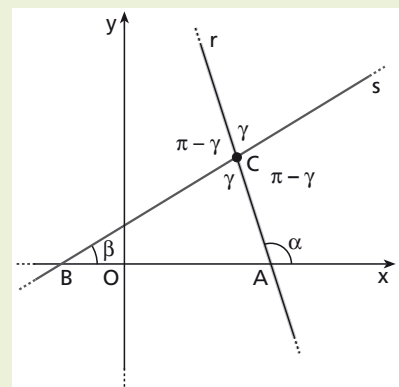
$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

L'angolo fra due rette

$$r : y = mx + q, \quad \text{con } m = \operatorname{tg} \alpha$$

$$s : y = m'x + q', \quad \text{con } m' = \operatorname{tg} \beta$$

$$\operatorname{tg} \gamma = \operatorname{tg}(\alpha - \beta) = \frac{m - m'}{1 + mm'}$$

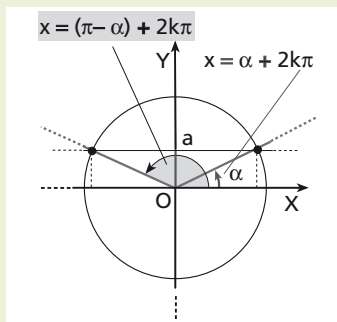


Equazioni goniometriche elementari

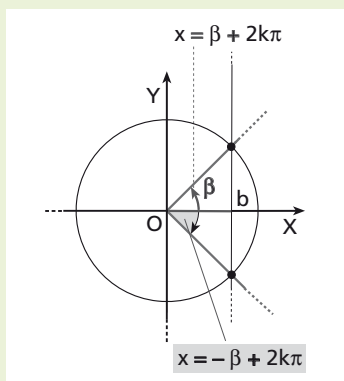
Un'equazione si dice **goniometrica** se contiene almeno una funzione goniometrica dell'incognita. Si chiamano **elementari** le equazioni goniometriche del tipo:

$\text{sen } x = a, \text{cos } x = b, \text{tg } x = c.$

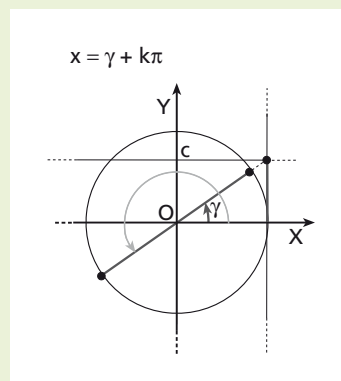
$\text{sen } x = a$
 determinata se $-1 \leq a \leq 1$
 impossibile se $a < -1 \vee a > 1$



$\text{cos } x = b$
 determinata se $-1 \leq b \leq 1$
 impossibile se $b < -1 \vee b > 1$



$\text{tg } x = c$ determinata $\forall c \in \mathbb{R}$



Ci sono particolari equazioni elementari che si possono risolvere con le proprietà della seguente tabella.

Tipo di equazione	Proprietà
$\text{sen } \alpha = \text{sen } \alpha'$	$\text{sen } \alpha = \text{sen } \alpha' \Leftrightarrow \alpha = \alpha' + 2k\pi \vee \alpha + \alpha' = \pi + 2k\pi$
$\text{sen } \alpha = -\text{sen } \alpha'$	$-\text{sen } \alpha' = \text{sen } (-\alpha')$
$\text{sen } \alpha = \text{cos } \alpha'$	$\text{cos } \alpha' = \text{sen} \left(\frac{\pi}{2} - \alpha' \right)$
$\text{sen } \alpha = -\text{cos } \alpha'$	$-\text{cos } \alpha' = -\text{sen} \left(\frac{\pi}{2} - \alpha' \right) = \text{sen} \left(-\frac{\pi}{2} + \alpha' \right)$
$\text{cos } \alpha = \text{cos } \alpha'$	$\text{cos } \alpha = \text{cos } \alpha' \Leftrightarrow \alpha = \pm \alpha' + 2k\pi$
$\text{cos } \alpha = -\text{cos } \alpha'$	$-\text{cos } \alpha' = \text{cos} (\pi - \alpha')$
$\text{tg } \alpha = \text{tg } \alpha'$	$\text{tg } \alpha = \text{tg } \alpha' \Leftrightarrow \alpha = \alpha' + k\pi$
$\text{tg } \alpha = -\text{tg } \alpha'$	$-\text{tg } \alpha' = \text{tg} (-\alpha')$

LE EQUAZIONI LINEARI IN SENO E COSENO

$$a \operatorname{sen} x + b \operatorname{cos} x + c = 0 \quad a \neq 0, b \neq 0$$

Metodo algebrico

- $c = 0 \rightarrow$ si divide per $\operatorname{cos} x \rightarrow \operatorname{tg} x = -\frac{b}{a}$.
- $c \neq 0 \rightarrow$ si determinano le eventuali soluzioni di tipo $x = \pi + 2k\pi$; se $x \neq \pi + 2k\pi$, applicando le formule parametriche si ottiene

$$\begin{cases} t^2(c-b) + 2at + b + c = 0 \\ t = \operatorname{tg} \frac{x}{2} \end{cases}$$

Metodo grafico

Si sostituisce $Y = \operatorname{sen} x$ e $X = \operatorname{cos} x$ e si risolve quindi il sistema seguente:

$$\begin{cases} X^2 + Y^2 = 1 \\ aY + bX + c = 0 \end{cases}$$

Metodo dell'angolo aggiunto

Si risolve il sistema seguente:

$$\begin{cases} \operatorname{sen}(x + \alpha) = -\frac{c}{r} \\ r = \sqrt{a^2 + b^2} \\ \operatorname{tg} \alpha = \frac{b}{a} \end{cases}$$

LE EQUAZIONI OMOGENEE DI SECONDO GRADO IN SENO E COSENO

$$a \operatorname{sen}^2 x + b \operatorname{cos} x \operatorname{sen} x + c \operatorname{cos}^2 x = 0$$

Primo metodo

- $a = 0 \rightarrow \operatorname{cos} x (b \operatorname{sen} x + c \operatorname{cos} x) = 0$
- $a \neq 0 \rightarrow$ si divide per $\operatorname{cos}^2 x \rightarrow a \operatorname{tg}^2 x + b \operatorname{tg} x + c = 0$

Secondo metodo

$$\text{Sostituendo} \begin{cases} \operatorname{sen} x \operatorname{cos} x = \frac{\operatorname{sen} 2x}{2} \\ \operatorname{sen}^2 x = \frac{1 - \operatorname{cos} 2x}{2} \\ \operatorname{cos}^2 x = \frac{1 + \operatorname{cos} 2x}{2} \end{cases} \text{ si ottiene un'equazione lineare.}$$

Un'equazione lineare della forma

$$a \operatorname{sen}^2 x + b \operatorname{sen} x \operatorname{cos} x + c \operatorname{cos}^2 x = d \quad (d \neq 0)$$

è riconducibile a un'equazione omogenea sostituendo $d = d(\operatorname{cos}^2 x + \operatorname{sen}^2 x)$.

Disequazioni goniometriche

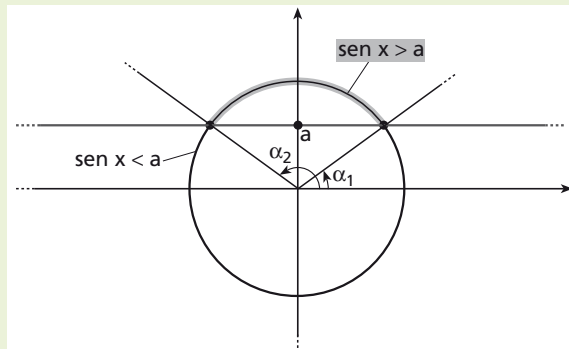
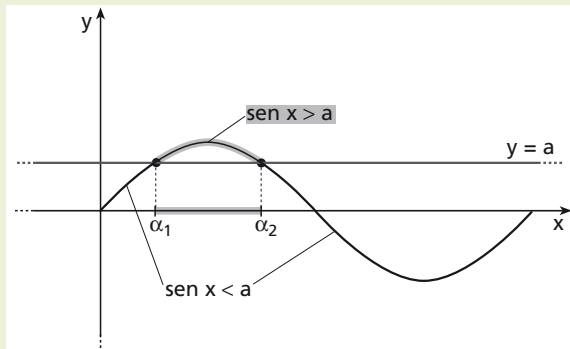
Primo metodo

Si studia la posizione reciproca tra il grafico della funzione goniometrica e la retta $y = a$.

Secondo metodo

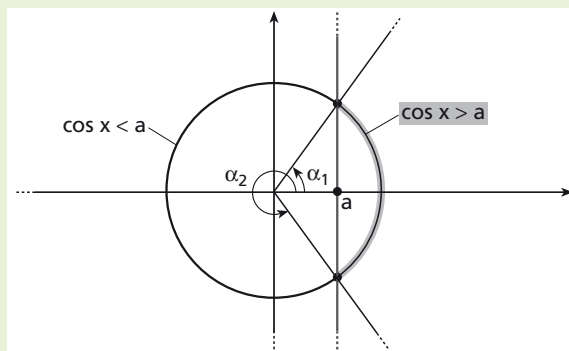
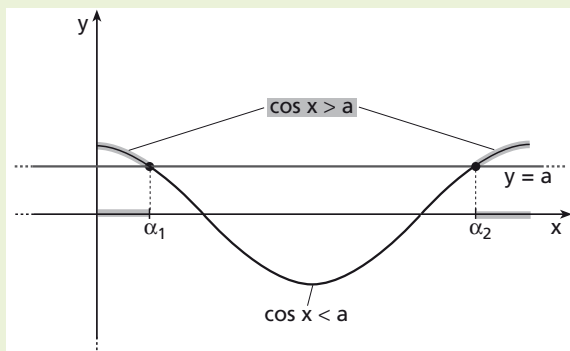
Si disegna la circonferenza goniometrica, si risolve l'equazione associata, si determinano gli archi in cui è soddisfatta.

La funzione seno



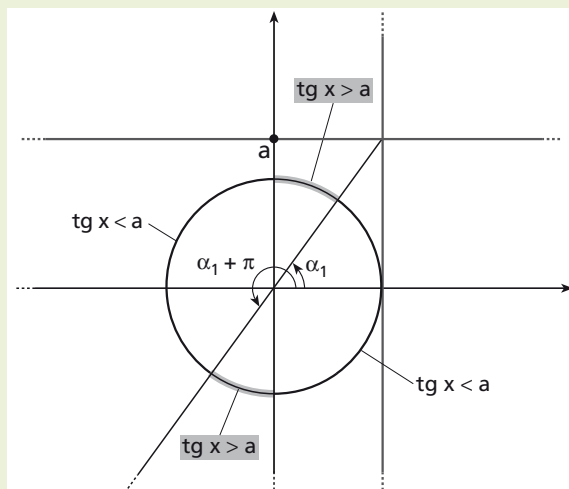
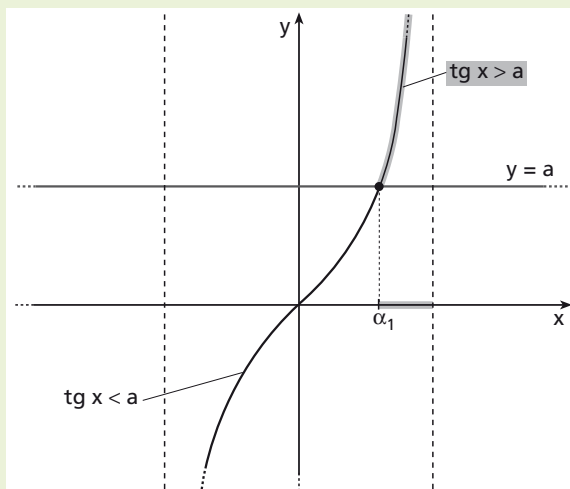
$\text{sen } x > a \rightarrow \alpha_1 + 2k\pi < x < \alpha_2 + 2k\pi$; $\text{sen } x < a \rightarrow 0 + 2k\pi < x < \alpha_1 + 2k\pi \vee \alpha_2 + 2k\pi < x < 2\pi + 2k\pi$

La funzione coseno



$\text{cos } x > a \rightarrow 0 + 2k\pi < x < \alpha_1 + 2k\pi \vee \alpha_2 + 2k\pi < x < 2\pi + 2k\pi$; $\text{cos } x < a \rightarrow \alpha_1 + 2k\pi < x < \alpha_2 + 2k\pi$

La funzione tangente



$\text{tg } x > a \rightarrow \alpha_1 + k\pi < x < \frac{\pi}{2} + k\pi$; $\text{tg } x < a \rightarrow -\frac{\pi}{2} + k\pi < x < \alpha_1 + k\pi$